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A TRIBUTE TO JAN KISYŃSKI

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In 1948, Einar Hille's enormous book, "Functional Analysis and Semi-Groups", was published by the American Mathematical Society [1]. The two most basic theorems of operator semigroup theory are:

Theorem 1. *For a linear operator $A : D(A) \subset X \to X$ with domain $D(A)$ dense in the Banach space X, the initial value problem*

$$du/dt = Au,\ u(0) = f \in D(A),\ t \geq 0$$

is wellposed (in the sense of existence, uniqueness and continuous dependence on f, A) if and only if it is solved by a strongly continuous (or class (C_0)) semigroup $T = \{T(t) : t \geq 0\}$ of bounded linear operators on X, and the solution is given by $u(t) = T(t)f$.

Theorem 2. *The initial value problem*

$$du/dt = Au, u(0) = f \in D(A),\ t \geq 0$$

is governed by a (C_0) semigroup $T = \{T(t) : t \geq 0\}$ if and only if A is the infinitesimal generator (i.e., $A = T'(0)$) of a (C_0) semigroup $T = \{T(t) : t \geq 0\}$ of bounded linear operators on X (and the solution is given by $u(t) = T(t)f$) if and only if A satisfies certain spectral conditions (which reduce to $\lambda \in \rho(A)$ and $||(\lambda I - A)^{-1}|| \leq 1/\lambda$ for all $\lambda > 0$ in case $||T(t)|| \leq 1$ for all $t \geq 0$, the contractive case).

Neither of these results were known when Hille started correcting his galley proofs in 1948. At that time Hille proved Theorem 2 in the contractive case, as did Kosaku Yosida, independently and simultaneously. Theorem 1 was proved in 1950 and 1951 by Hille and by Ralph Phillips (using different but equivalent defnitions of wellposedness). Hille invited Phillips, who had obtained major additional results, to expand Hille's book. It was done, and the Hille–Phillips book (of the same title) appeared in 1957 [2], with both additions and deletions from the 1948 version.

In 1971, Phillips asked me, during a summer research seminar on PDE at Stanford, which research problems was I studying. I mentioned some problems involving operator semigroups. Ralph responded that everything important about semigroups was known and contained in his 1957 book. This was of course wrong, as Phillips graciously admitted at a later date. In the 1960s and early 1970s were developed the approximation theorem (bicontinuous dependence of T on the generator A) for semigroups as well as new perturbation theory, a much better understanding of Richard Feynman's integral formula for the solution of the Schrödinger equation and the corresponding Feynman–Kac formula, much else in quantum theory and mathematical biology and Markov process theory, et al. One of the main contributors to all this and much more later was Jan Kisyński, whose work on approximation theory, cosine functions, the telegraph equation, Laplace transforms, and other topics were of major importance for both the theory and the applications. Moreover, nonlinear semigroup theory emerged starting in the late 1960s, and Kisyński's proofs were among the few that were the "right proofs" in that they extended nicely to the nonlinear context. Jan's brilliant ideas and wide scope continue to influence semigroup theory and applications today. It is a pleasure to thank Jan Kisyński on the occasion of his 85th birthday and to record the admiration of many, many mathematicians for his lasting contributions to semigroups through his brilliant research, his writings, his teaching, his influence, and his kindness and help to so many of us.

References

1. Hille E. *Functional Analysis and Semi-Groups.* N.Y., American Mathematical Society, 1948.
2. Hille E., Philips R.S. *Functional Analysis and Semi-Groups.* N.Y., American Mathematical Society, 1957.

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