MSC 76S05 DOI: 10.14529/mmp180410

MATHEMATICAL MODEL OF HEATING OF PLANE POROUS HEAT EXCHANGER OF HEAT SURFACE COOLING SYSTEM IN THE STARTING MODE

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Based on the conjugate Darcy–Brinkman–Forchheymer hydrodynamic model and Schumann thermal model with boundary conditions of the second kind, a model with lumped parameters was proposed by means of geometric 2D averaging to identify the integral kinetics of the temperature fields of a porous matrix and a Newtonian coolant without phase transitions. The model was adapted for a heat-stressed surface by means of a porous compact heat exchanger with uniform porosity and permeability, obeying the modified Kozeny–Carman relation, in the form of a Cauchy problem, the solution of which was obtained in the final analytical representation for the average volume temperatures of the coolant and the porous matrix. The possibility of harmonic damped oscillations of the temperature fields and the absence of coolant overheating in the starting condition of the cooling system were shown. For the dimensionless time of establishing the stationary functioning of the porous heat exchanger, an approximate estimate was obtained correlating with the known data of computational and full-scale experiments.

Keywords: flat porous heat exchanger; heat-stressed surface; boundary conditions of the second kind; time to settle a stationary warm regime.

Introduction

The classical use of porous materials for the intensification of single-phase heat-exchange processes [1] is greatly supplemented by new substantive applications, such as geothermal energy supply [2], technologies for obtaining structured polymeric materials [3], waste utilization in combustion in porous media [4], bioconvection in porous tissues of living organisms [5], etc. This requires consideration of transport phenomena in porous systems under nonstationary conditions [6].

The analysis of a wide range of phenomenological mathematical models of non-stationary hydrothermal fields in porous media [7] showed that the representation of the fundamental Darcy–Brinkman–Forchheymer and Schumann equations in the Hsu–Cheng form for the laminar flow regime of the Newtonian coolant [8]

$$\nabla \cdot \bar{v} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\left[\frac{\partial \bar{v}}{\partial \tau} + \frac{(\bar{v}\cdot\nabla)\,\bar{v}}{\varepsilon}\right] = -\nabla p + \mu_f \nabla^2 \bar{v} - \left[\mu_f \frac{\bar{v}}{K} + \rho_f \frac{b\bar{v}\,|\bar{v}|}{\sqrt{K}}\right], \tag{2}$$

$$\varepsilon\,(\rho c_p)_f\left(\frac{\partial t_f}{\partial \tau} + \bar{v}\nabla t_f\right) = \nabla\cdot\left(\lambda_e^f \cdot \nabla t_f\right) \pm \alpha_{sf} a_{sf}\,(t_s - t_f)\,, \tag{3}$$

$$(1-\varepsilon)\left(\rho c_p\right)_s \frac{\partial t_s}{\partial \tau}=\nabla\left(\lambda_e^s \cdot \nabla t_s\right) \mp \alpha_{sf} a_{sf}\left(t_s-t_f\right) \qquad (4)$$

is the most convincing when interpreting experimental data and meets the criteria of qualitative and quantitative adequacy, where τ is time; $(\rho, \mu, c_p)_f$ is density and dynamic viscosity of the liquid; ε is porosity; $(\rho, c_p)_s$ is density and heat capacity of the porous skeleton; $\bar{v}$ is a liquid velocity vector; $\bar{g}$ is a gravitational acceleration vector; p is pressure; K is a permeability of the medium; λ_e^f, λ_e^s are effective tensors of thermal conductivity coefficients of the liquid and the porous body skeleton material; t_f, t_s are temperature of the liquid and the skeleton of the porous body; α_{sf} is a coefficient of heat transfer between the liquid phase and the skeleton of the porous body; a_{sf} is a characteristic of surface area of the wetted surface in the porous medium.

For the first time the numerical analysis (1) – (4), under the assumption of homogeneity of the porous medium, constant thermophysical characteristics and local thermal equilibrium between the liquid phase and the porous skeleton, was given in [9] for the pulsational change in the liquid velocity at the entry to the porous layer with boundary thermal conditions of the second kind. The articles in [10, 11] are devoted to the experimental study of the settling of the temperature fields in porous media, in which the laws of the formation of the thermal initial region as a function of porosity were studied. However, the question related to the duration of the heating of the porous elements was not illuminated, in spite of the continuing interest in nonstationary processes in porous media [12].

1. Problem Statement and Synthesis of the Model

We consider a flat porous element in the $2-D$ format in the Cartesian coordinate system with the origin at the edge of the lower bounding plane, which is heat-generating with the heat flux density q_0. The longitudinal coordinate x is directed along the flow of the heat carrier (with known values of its velocity in the input section $u_0 = \text{const}$ and its temperature $t_0 = \text{const}$), the transverse coordinate is perpendicular to the heat-generating plane. The porous medium is bounded by the heat-insulating surface located parallel to the heat-generating plane at the distance h (the surfaces bounding the porous medium are impenetrable for the coolant). Under the same assumptions as in [9], but refusing to simplify $t_f = t_s$, the system of equations (1) – (4) in the dimensionless form can be written as [13]

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \qquad (5)$$

$$\frac{\partial U}{\partial \theta}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\left(\frac{1}{\mathrm{Re}\cdot\mathrm{Da}}+\frac{B}{\sqrt{\mathrm{Da}}}\sqrt{U^2+V^2}\right)U, \qquad (6)$$

$$\frac{\partial V}{\partial \theta}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\left(\frac{1}{\mathrm{Re}\cdot\mathrm{Da}}+\frac{B}{\sqrt{\mathrm{Da}}}\sqrt{U^2+V^2}\right)V, \qquad (7)$$

$$\frac{1}{\varepsilon}\frac{\partial T_f}{\partial \theta}+U\frac{\partial T_f}{\partial X}+V\frac{\partial T_f}{\partial Y}=\frac{1}{\mathrm{Pr}\cdot\mathrm{Pe}}\left(\frac{\partial^2 T_f}{\partial X^2}+\frac{\partial^2 T_f}{\partial Y^2}\right)+\frac{\mathrm{Nu}_p\cdot\mathrm{Re}}{\mathrm{Pr}\cdot\mathrm{Re}_p^2}\left(\Lambda T_s-T_f\right), \qquad (8)$$

$$(1-\varepsilon)\,\mathrm{Lu}\cdot\mathrm{Pr}\cdot\mathrm{Re}\,\frac{\partial T_s}{\partial \theta}=\frac{\partial^2 T_s}{\partial X^2}+\frac{\partial^2 T_s}{\partial Y^2}-\mathrm{Nu}_p\cdot\left(\frac{\mathrm{Re}}{\mathrm{Re}_p}\right)^2\left(\Lambda T_s-T_f\right), \qquad (9)$$

where $\theta = u_0\tau/(\varepsilon h)$; $X = x/h$; $Y = y/h$; $U = u/u_0$; $V = v/u_0$; u, v are components of the liquid velocity vector $\bar{v}$; $P = \varepsilon^2 p/(\rho_f u_0^2)$; $B = \varepsilon^2 b$; b is the Forchheymers factor; $T_f = \lambda_e^f(t_f - t_0)/(q_0 h)$; $T_s = \lambda_e^s(t_s - t_0)/(q_0 h)$; $\Lambda = \lambda_e^f/\lambda_e^s$; $\mathrm{Re} = p_f u_0 h/(\mu_f \varepsilon^2)$ is Reynolds number; $\mathrm{Re}_p = \rho_f u_0 d_p/[6(1-\varepsilon)\mu_f]$ is pore Reynolds number; $\mathrm{Da} = K/h^2$ is Darcy number; $\mathrm{Pr} = \varepsilon(\rho c_p)_f \mu_f\big/\left(\lambda_e^f \rho_f\right)$ is Prandtl number; $\mathrm{Nu}_p = \alpha_{sf} d_p/\lambda_e^f$ is pore Nusselt number; d_p is a characteristic size of the extraporate space; $\mathrm{Lu} = \left[\lambda_e^f\big/(\rho c_p)_f\right]\big/\left[\lambda_e^s/(\rho c_p)_s\right]$ is modified Lykov criterion characterizing the diffusion of heat in the liquid with respect to the diffusion of heat in the skeleton of a porous medium.

The hypothesis of a unidirectional flow of a coolant in the laminar regime [13], taking into account the fact that the diffusion of heat in the transverse direction of the porous layer is substantially greater than in the longitudinal one $\left(\partial T_{f,s}^2/\partial Y^2 \gg \partial T_{f,s}^2/\partial X^2\right)$[14], allows to write the system (5) – (9) in the simplified form

$$\frac{1}{\varepsilon}\frac{\partial T_f}{\partial \theta} + \frac{\partial T_f}{\partial X} = \frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\frac{\partial T_f^2}{\partial Y^2} + \frac{\mathrm{Nu}_p Re}{\mathrm{Re}_p^2\cdot\mathrm{Pr}}(\Lambda T_s - T_f), \tag{10}$$

$$(1-\varepsilon)\,\mathrm{Lu}\;\mathrm{Re}\cdot\mathrm{Pr}\frac{\partial T_s}{\partial \theta} = \frac{\partial^2 T_s}{\partial Y^2} - \mathrm{Nu}_p\left(\frac{\mathrm{Re}}{\mathrm{Re}_p}\right)^2(\Lambda T_s - T_f) \tag{11}$$

with the thermal boundary conditions corresponding to the formulation of the problem

$$T_f(X,Y,0) = T_s(X,Y,0) = 0, \tag{12}$$

$$\frac{\partial T_f(X,0,\theta)}{\partial Y} = \frac{\partial T_s(X,0,\theta)}{\partial Y} = -1, \tag{13}$$

$$\frac{\partial T_f(X,1,\theta)}{\partial Y} = \frac{\partial T_s(X,1,\theta)}{\partial Y} = 0. \tag{14}$$

When $\mathrm{Da} \to 0$, which corresponds to the maximum possible values of the heat transfer interphase [15], the liquid velocity profile in the porous layer is close to the ideal displacement regime, i.e. $U \approx 1$, which leads to the further simplification of system (10) – (14), since there is no need to solve the hydrodynamic problem.

The transition from the model with distributed parameters (10) – (14) to the model with lumped parameters is carried out according to the rule of geometric averaging

$$\overline{\overline{T}}_{f,s}(\theta) = \frac{1}{L}\int_0^1\int_0^L T_{f,s}(X,Y,\theta)\,dX\,dY,$$

where $L = l/h$, l is a length of the porous heat exchanger. As a result, we obtain the Cauchy problem for the system of ordinary differential equations of the first order

$$\frac{d\overline{\overline{T}}_f}{d\theta} = \frac{\varepsilon}{\mathrm{Pr}\,\mathrm{Re}} + \varepsilon\frac{\mathrm{Nu}_p\,\mathrm{Re}}{\mathrm{Pr}\,\mathrm{Re}_p^2}\Lambda\overline{\overline{T}}_s - \left(\frac{\varepsilon}{L} + \varepsilon\frac{\mathrm{Nu}_p\,\mathrm{Re}}{\mathrm{Pr}\,\mathrm{Re}_p^2}\right)\overline{\overline{T}}_f, \tag{15}$$

$$\frac{d\overline{\overline{T}}_s}{d\theta} = \frac{1}{(1-\varepsilon)\,\mathrm{Lu}\;\mathrm{Pr}\,\mathrm{Re}} - \frac{\mathrm{Nu}_p\,\mathrm{Re}}{(1-\varepsilon)\,\mathrm{Lu}\;\mathrm{Pr}\,\mathrm{Re}_p^2}\Lambda\overline{\overline{T}}_s + \frac{\mathrm{Nu}_p\,\mathrm{Re}}{(1-\varepsilon)\,\mathrm{Lu}\;\mathrm{Pr}\,\mathrm{Re}_p^2}\Lambda\overline{\overline{T}}_f, \tag{16}$$

$$\overline{\overline{T}}_s(0) = \overline{\overline{T}}_f(0) = 0. \tag{17}$$

2. Closing Relations

The classical physical model of porous media, as a rule, is presented in the form of dense packing of spheres [16], the voids of which are interconnected and completely filled with liquid, with only two phases: a liquid and a porous non-deformable skeleton. As the thermophysical parameters in (15) – (17) are homogeneous in space coordinates and do not depend on the temperature, they can be calculated from the relations [17]:

$$a_{sf} = 6\left(1-\varepsilon\right)/d_p, \tag{18}$$

$$\alpha_{sf} = \lambda_f \left[2 + 1,1 \mathrm{Pr}^{0^{1/3}} \left(\rho_f u_0 d_p/\mu_f\right)^{0,6}\right]/d_p, \tag{19}$$

where $\mathrm{Pr}^0 = \mu_f c_{pf}/\lambda_f$; λ_f is heat conductivity of the liquid;

$$\lambda_e^f = \left[\varepsilon + \left(0,1 \div 0,5\right)\mathrm{Pr}^{0^{1/3}}\left(\rho_f u_0 d_p/\mu_f\right)\right]\lambda_f, \tag{20}$$

$$\lambda_e^s = \left(1-\varepsilon\right)\lambda_s, \tag{21}$$

where λ_s is heat conductivity of the skeleton of the porous medium; d_p is number-average diameter of spherical particles in the porous layer. The dimensionless parameters in (15) – (17) on the basis of (18) – (21) can be defined as follows: $\mathrm{Re} = \mathrm{Re}^{\,0}/\varepsilon^2$, $\mathrm{Re}_{\,p} = \mathrm{Re}^{\,0}_{\,p}/\left[6\left(1-\varepsilon\right)\right]$, $\mathrm{Pr} = \varepsilon\,\mathrm{Pr}^0/\left(\varepsilon + 0,3\,\mathrm{Pr}^0\,\mathrm{Re}^{\,0}_{\,p}\right)$, $\mathrm{Nu}_{\,p} = \left(2 + 1,1\mathrm{Pr}^{0^{1/3}}\mathrm{Re}_{\,p}^{\,3/5}\right)/\left(\varepsilon + 0,3\mathrm{Pr}^0\mathrm{Re}^0_{p}\right)$, where $\mathrm{Re}^{\,0} = \rho_f u_0 h/\mu_f$, $\mathrm{Re}^{\,0}_{\,p} = \rho_f u_0 d_p/\mu_f$.

3. Solutions

The change in the temperatures of the coolant and the skeleton of the porous medium from the dimensionless time is obtained by means of the one-sided integral Laplace transform [18]

$$\left[\gamma\Lambda B + \varepsilon\left(L^{-1}+B\right)\right]^2 < 4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right),$$

$$\overline{\overline{T}}_f(\theta) = \varepsilon A\left\{\frac{1}{\omega}\left[\frac{\left(a+\gamma\Lambda B\right)^2+\omega^2}{a^2+\omega^2}\right]^{0,5}\exp\left(a\theta\right)\sin\left(\omega\theta + \mathrm{arctg}\,\frac{\omega}{a+\gamma\Lambda B} - \mathrm{arctg}\,\frac{\omega}{a}\right) + \right.$$
$$\left. + \frac{\gamma\Lambda B}{a^2+\omega^2}\right\} + \varepsilon\gamma\Lambda AB\left[\frac{1}{\omega}\left(\frac{1}{a^2+\omega^2}\right)^{0,5}\exp\left(a\theta\right)\sin\left(\omega\theta - \mathrm{arctg}\,\frac{\omega}{a}\right) + \frac{1}{a^2+\omega^2}\right],$$

$$\overline{\overline{T}}_s(\theta) = \gamma A\left\langle\frac{1}{\omega}\left\{\frac{\left[a+\varepsilon\left(L^{-1}+B\right)\right]^2+\omega^2}{a^2+\omega^2}\right\}^{0,5}\exp\left(a\theta\right)\sin\left(\omega\theta + \mathrm{arctg}\,\frac{\omega}{a+\varepsilon\left(L^{-1}+B\right)} - \mathrm{arctg}\,\frac{\omega}{a}\right) + \right.$$
$$\left. + \frac{\varepsilon\left(L^{-1}+B\right)}{a^2+\omega^2}\right\rangle + \varepsilon\gamma AB\left[\frac{1}{\omega}\left(\frac{1}{a^2+\omega^2}\right)^{0,5}\exp\left(a\theta\right)\sin\left(\omega\theta - \mathrm{arctg}\,\frac{\omega}{a}\right) + \frac{1}{a^2+\omega^2}\right],$$

when $\left[\gamma\Lambda B + \varepsilon\left(L^{-1}+B\right)\right]^2 = 4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right)$,

$$\overline{\overline{T}}_f(\theta) = \frac{2\varepsilon\gamma\Lambda AB}{s_0^2}\left\{1 + \left[-1 + s_0\left(1 + \frac{s_0}{2\gamma\Lambda B}\right)\theta\right]\exp\left(s_0\theta\right)\right\},$$
$$\overline{\overline{T}}_s(\theta) = \frac{\varepsilon\gamma A\left(L^{-1}+2B\right)}{s_0^2}\left\{1 + \left[-1 + s_0\left(1 + \frac{s_0}{\varepsilon\left(L^{-1}+2B\right)}\right)\theta\right]\exp\left(s_0\theta\right)\right\},$$

when

$$\left[\gamma\Lambda B+\varepsilon\left(L^{-1}+B\right)\right]^{2}>4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right), \tag{22}$$

$$\overline{\overline{T}}_{f}\left(\theta\right)=\frac{2\varepsilon\gamma\Lambda AB}{s_{1}s_{2}}+\frac{\varepsilon As_{1}+2\varepsilon\gamma\Lambda AB}{s_{1}\left(s_{1}-s_{2}\right)}\exp\left(s_{1}\theta\right)+\frac{\varepsilon As_{2}+2\varepsilon\gamma\Lambda AB}{s_{2}\left(s_{2}-s_{1}\right)}\exp\left(s_{2}\theta\right), \tag{23}$$

$$\overline{\overline{T}}_{s}\left(\theta\right)=\frac{2\varepsilon\gamma A\left(L^{-1}+2B\right)}{s_{1}s_{2}}+\frac{\gamma As_{1}+\varepsilon\gamma A\left(L^{-1}+2B\right)}{s_{1}\left(s_{1}-s_{2}\right)}\exp\left(s_{1}\theta\right)+$$

$$+\frac{\gamma As_{2}+\varepsilon\gamma A\left(L^{-1}+2B\right)}{s_{2}\left(s_{2}-s_{1}\right)}\exp\left(s_{2}\theta\right), \tag{24}$$

where $A=1/\left(\mathrm{Pr}\cdot\mathrm{Re}\right)$; $B=\mathrm{Nu}_{p}\mathrm{Re}/\left(\mathrm{Pr}\cdot\mathrm{Re}_{p}^{2}\right)$, $\gamma=1/\left[\left(1-\varepsilon\right)\mathrm{Lu}\right]$, $\omega=\sqrt{4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right)-\left[\gamma\Lambda B+\varepsilon\left(L^{-1}+B\right)\right]^{2}}$, $s_{0}=-\left[\gamma\Lambda B+\varepsilon\left(L^{-1}+B\right)\right]/2$,

$$s_{1}=s_{0}+\sqrt{s_{0}^{2}-4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right)},\ s_{2}=s_{0}-\sqrt{s_{0}^{2}-4\varepsilon\gamma\Lambda B\left(L^{-1}+2B\right)}.$$

4. Computing Experiment

Let's consider the real situation. Let the porous layer consists of dense packing of identical copper ball elements with the diameter $d_p=0,5\cdot10^{-3}$m, and $\varepsilon=0,4$; $h=0,01$m; $l=0,02$m. Thermophysical characteristics of the coolant are close to water, then $\rho_f=1000$ kg/m^3; $\rho_s=2700$ kg/m^3; $\mu_f=5\cdot10^{-4}$ Pa·s; $c_{\rho f}=4190$ J/(kg·K); $c_{ps}=880$J/(kg·K); $\lambda_f=0,68$ W/(m·K); $\lambda_s=211$ W/(m·K). The values of the determining parameters are given in Table.

Table

Re_0	Pr	Re_p	Re	Lu	Λ	Nu_p
20	0,930	0,278	125	0,004	0,007	2,719
100	0,245	1,389	625	0,015	0,027	1,235
200	0,128	2,778	1250	0,029	0,052	0,868

The dimensionless form of recording the equations of the mathematical model allows to abstract from the specific value of the heat flux released by the cooled heat-stressed surface. The results of the calculations show (Fig. 1) that with an increase in the Reynolds number, the dimensionless time of establishment of the stationary regime increases. However, in denominate quantity, it decreases, as value u_0 grows faster than θ. This means that a larger flow of the coolant through a porous element leads to an early onset of a stationary regime, which is consistent with physical concepts of heat transfer in porous media.

The local maximum of the temperature of the porous skeleton is explained by the fact that the coolant is present in the region adjacent to the inlet for some time. At the same time, the heat comes from the heat-generating surface by the mechanism of thermal conductivity into the skeleton itself, which is still free from the coolant, which causes the increase in its temperature. With the complete passage of the coolant through the porous element, the temperature of the skeleton decreases and approaches the stationary value. For the coolant temperature, there is no such a regularity. Thus, the proposed model makes it possible to answer the question of the impossibility of the impact of overheating

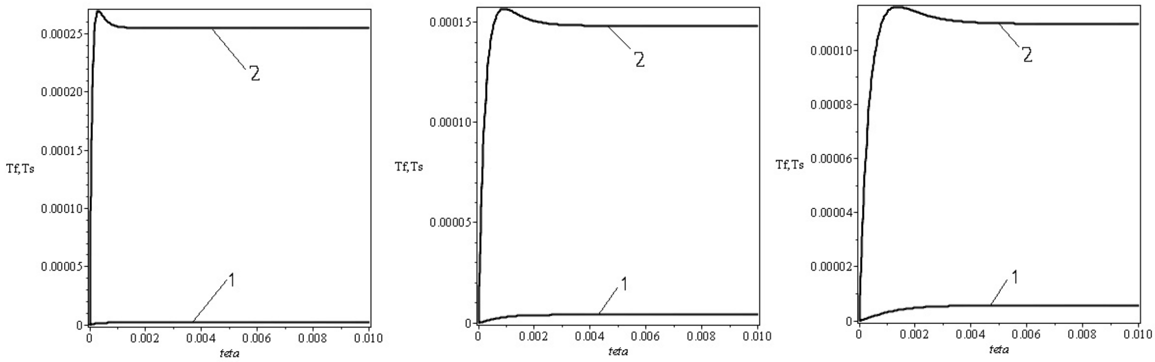

Fig. 1. Dynamics of the temperatures of the coolant and the porous heat-exchange element when the heat-stressed surface is cooled for different Reynolds numbers of the inlet coolant flow: a – 20; b – 2; c – 5; 1 – T_f; 2 – T_s

of the porous skeleton in the starting condition on the creation of the conditions for the phase transition in the coolant.

The increase in porosity (Fig. 2) leads to the increase in the dimensionless and real transition time, since in this case, the Reynolds number for the interskeleton space decreases and in addition the transition temperature naturally increases.

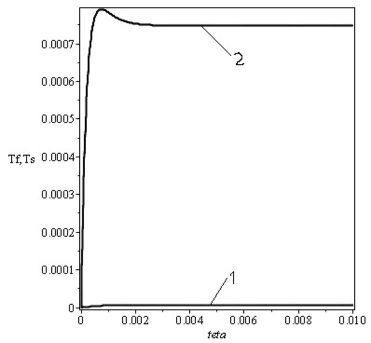

Fig. 2. Dynamics of the temperatures of the coolant and the porous heat exchange element at Re $_0 = 20$ and $\varepsilon = 0,5$: 1 – T_f; 2 – T_s

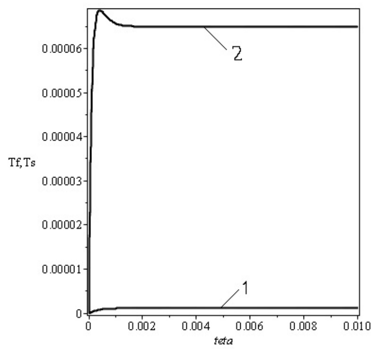

Fig. 3. Dynamics of the temperatures of the coolant and the porous heat exchange element at Re $_0 = 200$ and $d_p = 3 \cdot 10^{-5}$: 1 – T_f; 2 – T_s

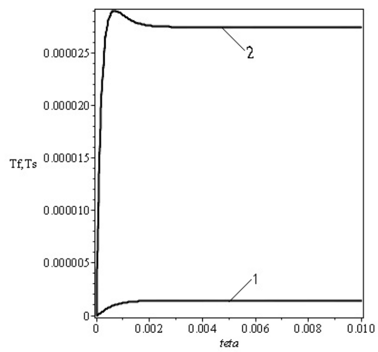

Fig. 4. Dynamics of the temperatures of the coolant and the porous heat exchange element at Re $_0 = 200$ and $h = 0,02m$: 1 – T_f; 2 – T_s

Reducing the dispersion of the particles in the porous element shortens the transition time and reduces the temperature of the coolant and the porous skeleton by increasing the surface area of the heat transfer (Fig. 3).

The increase in the thickness of the porous layer leads to the decrease in the temperature of the coolant and the porous skeleton with the reduction in the transition time (Fig. 4).

Since, for the vast majority of practically important cases, condition (22) is satisfied and s_1 is substantially smaller than s_2, then, for example, from (24) we can find an approximate estimate of the dimensionless time for setting the stationary cooling regime from

$$\overline{\overline{T_s}}\left(\theta_c\right) \Big/ \overline{\overline{T_s}}\left(\infty\right) - 1 = \delta,$$

from where

$$\theta_c \approx \frac{1}{s_1} \ln\left[\frac{2\delta\left(s_1 - s_2\right)\gamma\Lambda B}{s_2\left(s_1 + 2\gamma\Lambda B\right)}\right], \qquad (25)$$

where δ is predetermined relative accuracy of determination θ_c (usually $\delta = 0,01$).

Proceeding from the assumption that the thermal initial section in the flat porous layer in the hydrodynamic regime of the ideal displacement of the coolant with constant velocity is formed in time similarly to the heat transfer coefficient along the axial coordinate, comparison with the experimental data from [19] (Fig. 5) shows a satisfactory correlation with the calculation results according to the proposed formula (25).

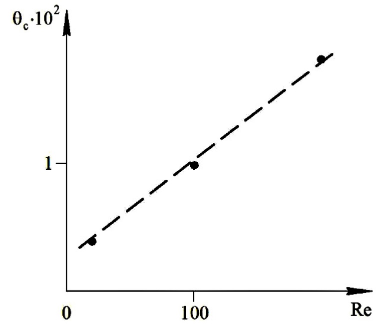

Fig. 5. Dependence of the determination of the dimesionless time on the Reynolds number for the coolant: $---$ is empirical approximation [19]; • is calculation by (25)

Conclusion

The proposed model makes it possible to identify the duration of the transient thermal regime when cooling heat-stressed surface with a porous heat exchanger without complicated calculations and to not only select the rational macro- and micro-geometry of the porous skeleton, but also to evaluate the effect of thermophysical characteristics on the kinetics of the setting of the thermal stationary regime.

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Received July 16, 2018

УДК 621.1.016.4(03) DOI: 10.14529/mmp180410

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ПРОГРЕВА ПЛОСКОГО ПОРИСТОГО ТЕПЛООБМЕННИКА СИСТЕМЫ ОХЛАЖДЕНИЯ ТЕПЛОВЫДЕЛЯЮЩЕЙ ПОВЕРХНОСТИ В РЕЖИМЕ ПУСКА

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На основе сопряженных гидродинамической модели Дарси – Бринкмана – Форчхеймера и тепловой модели Шуманна с граничными условиями второго рода путем геометрического 2D-осреднения предложена модель с сосредоточенными параметрами для идентификации интегральной кинетики температурных полей пористой матрицы и ньютоновского теплоносителя без фазовых переходов. Модель адаптирована для охлаждения теплонапряженной поверхности с помощью пористого теплообменника с однородной пористостью и проницаемостью, подчиняющейся модифицированному соотношению Козени – Кармана, в виде задачи Коши, решение которой получено в конечном аналитическом представлении для среднеобъемных температур теплоносителя и пористой матрицы. Показана возможность существования гармонического затухающего колебания полей температур и отсутствие перегрева теплоносителя в пусковом режиме системы охлаждения. Для безразмерного времени установления стационарного функционирования пористого теплообменника получена приближенная оценка, коррелирующая с известными данными вычислительного и натурного экспериментов

Ключевые слова: плоский пористый теплообменник; теплонапряженная поверхность; граничные условия второго рода; пусковой режим; время установления стационарного теплового режима.

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Поступила в редакцию 16 июля 2018 г.