MSC 37C75, 37N40, 34D23 DOI: 10.14529/mmp190302

GLOBAL SCHUMPETERIAN DYNAMICS WITH STRUCTURAL VARIATIONS

***A.N. Kirillov*[1], *A.M. Sazonov*[1]**
[1]Institute of Applied Mathematical Research of the Karelian Research Centre of the Russian Academy of Sciences, Petrozavodsk, Russian Federation
E-mails: krllv1812@yandex.ru, sazonov@cs.karelia.ru

In this paper, we present the investigations developing the schumpeterian theory of endogenous evolution of economic systems. The proposed approach allows to simulate the emergence and propagation of new technologies. We develop a mathematical model of dynamics of sector capital distribution over efficiency levels on the base of the system of nonlinear differential equations. In order to take into account the boundedness of the economic growth conditioned by the boundedness of the markets, the resource base and other factors, we introduce the notion of economical niche volume. The scenario of the emergence of the new highest efficiency level is proposed. In order to simulate the process of the emergence of the new highest efficiency level, the notion of intellectual capital is proposed. According to the proposed scenario, the new level emerges when the intellectual capital achieves the threshold value. Herewith, the dimension of the dynamic system is varied. The necessary condition for the functioning of the new level is formulated. The invariant set of the dynamic system is defined. The local stability of the equilibria is investigated. The global stability of the dynamic system is established on the base of a geometrical method. The proposed models allow to evaluate and predict the dynamics of the technological levels of the economic sector firms development.

Keywords: dynamic systems; Schumpeterian dynamics; stability.

Introduction

J. Schumpeter, the eminent austrian economist, proposed the theory of endogenous economic growth [1]. According to J. Schumpeter, the economic growth is based on the following two processes: creation of new technologies, i.e. innovation, and borrowing of new technologies, i.e. imitation. The conception of creative destruction and discrete evolution proposed by J. Schumpeter is developed in modern economic theory (for example, see D. Acemoglu, D. Cao [2], C. Christensen, M. Raynor [3], S.Yu. Glazev [4], P. Klimek, R. Hausmann, S. Thurner [5], etc.).

K. Iwai gives the first mathematical model of the theory proposed by J. Schumpeter [6, 7]. The further development of this theory is presented in investigations by V.M. Polterovich and G.M. Khenkin [8–10]. They proposed the evolution equation describing dynamics of the firms distribution over efficiency levels. Let $F_i(t)$ be the share of firms corresponding to the levels, which numbers are less or equal to i at the moment of time t. Then the evolution equation is as follows:

$$\dot{F}_i = \frac{dF_i}{dt} = -(\alpha + \beta(1 - F_i(t)))(F_i(t) - F_{i-1}(t)), i = 1, 2, ...,$$

with the boundary and initial conditions

$$F_0(t) \equiv 0, 0 < F_{i-1}(0) < F_i(0) < 1 \text{ if } 1 < i < N, F_i(0) = 1 \text{ if } i \geqslant N,$$

where N is the initial number of levels for $t = 0$, the terms $-\alpha(F_i - F_{i-1})$ and $-\beta(1 - F_i)(F_i - F_{i-1})$ describe the innovation and imitation processes, respectively, while $\alpha > 0$ is the innovation rate, and $\beta > 0$ is the imitation rate.

Note that this model was successfully applied in order to analyze the dynamics of the ferrous metal industry firms distribution over efficiency levels (L.M. Gelman, M.I. Levin, V.M. Polterovich, V.A. Spivak [8]).

Further, A.A. Shananin and G.M. Khenkin considered the mathematical model of the firm capacity dynamics [11]. Let $M_i(t)$ be the integrated firm capacity at the i-th level, λ_i be the profit per capacity unit at the i-th level, and $0 \leqslant \varphi_i(t) \leqslant 1$ be the share of the investments of the firms at the i-th level to create the capacities at the next ($i+1$-th) level. Then the capacity dynamics equation is as follows:

$$\dot{M}_i = (1-\varphi_i)\lambda_i M_i + \varphi_{i-1}\lambda_{i-1}M_{i-1}, i = 1, 2, \dots, \qquad (1)$$

with the boundary and initial conditions

$$M_0(t) \equiv 0, M_i(0) \geqslant 0, \sum_{i=1}^{N} M_i(0) > 0, M_i(0) = 0 \text{ if } i > N,$$

where N is the initial number of levels, $\varphi_i = \alpha + \beta(1 - \frac{\sum_{k=0}^{i} M_k}{\sum_{k=0}^{\infty} M_k})$, $\alpha > 0$, $\beta > 0$ are constants, $F_i(t) = \frac{\sum_{k=0}^{i} M_k}{\sum_{k=0}^{\infty} M_k}$.

However, this model does not take into account the boundedness of the economic growth conditioned by the boundedness of the markets, the resource base and other factors. Let us show that $M_i(t) \to \infty$ for $t \to \infty$.

First of all, consider the first equation ($i = 1$):

$$\dot{M}_1 = (1-\varphi_1)\lambda_1 M_1.$$

Substitute the expression of φ_1 and obtain

$$\dot{M}_1 = (1 - \alpha - \beta + \beta\frac{M_1(t)}{\sum_{k=0}^{\infty} M_k(t)})\lambda_1 M_1.$$

Lemma 1. *If $0 \leqslant \varphi_i(t) \leqslant 1$, then $\alpha + \beta < 1$.*

Proof. First of all, we prove that $F_1(t) \to 0$ for $t \to \infty$. According to Theorem 2 [11], $|F_1(t) - F_1^*(t)| \to 0$ for $t \to \infty$, where $F_1^*(t) = \left(1 + A(\frac{\alpha}{\alpha+\beta})^n e^{\beta t}\right)^{-1}$. Therefore, for any $\varepsilon > 0$, there exists $T_1 > 0$ such that $|F_1^*(t) - F_1(t)| < \frac{\varepsilon}{2}$ for $t > T_1$. Obviously, $F_1^*(t) \to 0$ for $t \to \infty$, i.e. for the same $\varepsilon > 0$ there exists $T_2 > 0$ such that $F_1^*(t) < \frac{\varepsilon}{2}$ for $t > T_2$. Consequently, for any $\varepsilon > 0$ there exists $T = max(T_1, T_2) > 0$ such that $F_1(t) < F_1^*(t) + |F_1^*(t) - F_1(t)| < \varepsilon$ for $t > T$, i.e. $F_1(t) \to 0$ for $t \to \infty$.

Now we assume that $\alpha + \beta \geqslant 1$. Then

$$0 \leqslant 1 - \varphi_1(t) = 1 - \alpha - \beta + \beta F_1(t) < 1 - \alpha - \beta + \beta\varepsilon \leqslant 1, \text{ for } t > T.$$

If $0 < \varepsilon < \dfrac{\alpha+\beta-1}{\beta}$, then $1 - \varphi_1 < 1 - \alpha - \beta + \beta\varepsilon < 0$ that contradicts $0 \leqslant \varphi_i(t) \leqslant 1$. Therefore, $\alpha + \beta < 1$. □

Note that there exists $\sigma > 0$ such that $\sigma < 1-\alpha-\beta < 1-\alpha-\beta+\beta\frac{M_1(t)}{\sum\limits_{k=0}^{\infty} M_k(t)}$, since $0 < \alpha+\beta < 1$. Therefore, we have

$$\dot{M}_1 = (1-\alpha-\beta+\beta\frac{M_1(t)}{\sum\limits_{k=0}^{\infty} M_k(t)})\lambda_1 M_1 > \sigma\lambda_1 M_1.$$

Integrate this inequality on the segment $[0,t]$ and obtain

$$M_1 > M_1^0 e^{\sigma\lambda_1 t} \to \infty, \text{for } t \to \infty,$$

where $M_1^0 = M_1(0)$. Therefore, $M_1(t) \to \infty$ for $t \to \infty$.

Taking into account (1), for $i \geqslant 2$, we obtain

$$\dot{M}_i = (1-\varphi_i)\lambda_i M_i + \varphi_{i-1}\lambda_{i-1}M_{i-1} > (1-\varphi_i)\lambda_i M_i.$$

Similar to the case $i = 1$, since $\varphi_i = \alpha + \beta(1 - \frac{\sum\limits_{k=0}^{i} M_k}{\sum\limits_{k=0}^{\infty} M_k})$, we obtain that $M_i(t) \to \infty$ for $t \to \infty$.

In order to construct the mathematical models of the dynamics of capital distribution over efficiency levels, we develop the approach proposed by A.A. Shananin and G.M. Khenkin [11]. In contrast with model (1), we consider φ_i to be constants. In our model, we take into account the boundedness of the economic growth. To this end, we introduce the notion of the economic niche volume, which is analogous to the notion of the ecological niche volume. By the economic niche volume we mean a limit integrated capital value, for which the growth rate is so low that there is no capital growth. Moreover, based on the dynamic system with variable dimension, we propose an approach to simulate the process of new levels emergences. The equilibria of the constructed dynamic models are determined and their global stability is proved. This research develops the previous results of the authors [12]. To this end, we introduce the structural variations based on the system with variable number of efficiency levels, which is controlled by the intellectual capital dynamics.

1. Model of Dynamics of Sector Capital Distribution Over Efficiency Levels

Following the approach proposed by V.M. Polterovich and G.M. Khenkin, we assume that the capital at each i-th efficiency level is distributed in two flows: to develop the production process at the next ($i+1$-th) level, and to produce at the current (i-th) level. The firms of the highest level make investments to create the new level that did not exist previously. These investments are distributed for the scientific research and for the capital accumulation, which will be used at the startup stage of the new efficiency level functioning.

We define the intellectual capital as the financial accumulations, which the firms at the highest level invest in scientific research and introduction of the research results into the industry.

Let $C_i(t)$ be the integrated capital at the i-th level (the same firm can have the capital at different levels), V_i be the economic niche volume at the i-th level, φ_i be the share of the firms capital at the i-th level intended for developing of the production at the next ($i+1$-th) level, λ_i be the unit prime cost at the i-th level (i.e. the cost of the unit of the goods production per unit time), $i = 1,..,N$, I be the intellectual capital, ν be the rate of the scientific research, ψ be the share of the investments intended for the scientific research. The rest capital is intended for the creation of the new $N+1$-th level (as initial capital). All values above are constants with the exclusion of $C_i(t)$. We assume that the greater i the higher the level.

Consider the system of differential equations

$$\begin{cases} \dot{C}_1 = \frac{1-\varphi_1}{\lambda_1} C_1(V_1 - C_1) = f_1, \\ \dot{C}_i = \frac{1-\varphi_i}{\lambda_i} C_i(V_i - C_i) + \varphi_{i-1} C_{i-1}, i = 2,..,N = f_i, \\ \dot{I} = -\nu I + \psi \varphi_N C_N = g. \end{cases} \tag{2}$$

Obviously, at each level of efficiency (with the exclusion of the first level), the right hand side of the equation of the capital growth rates contains two terms, which describe the capital growth provided by the production at the current (i-th) level and the investments from the previous ($i-1$-th) level, respectively. If the capital volume is not greater than the economic niche volume of the current level, then the first term is positive, i.e. the capital increases. Otherwise, the first term is negative, i.e. the production is unprofitable at the current level. However, the investments from the previous level provide the capital growth at the current level for some time, while the positive second term is greater than the absolute value of the negative first term, i.e. while the investments from the previous ($i-1$-th) level are greater than the firm losses induced by exceeding of the economic niche volume at the current (i-th) level.

The last equation describes the intellectual capital dynamics and is obtained as follows. The first negative term $-\nu I$ corresponds to the decrease in the intellectual capital I due to its spending for the scientific research conducted with the rate ν. The second term describes the growth of the intellectual capital provided by the science investments of the firms at the highest level. Let us consider the growth of the intellectual capital I during the time interval $[t, t+\Delta t]$. The value $\nu I(t)\Delta t$ is spent on the scientific research, while the investments obtained from the highest level are equal to $\psi\varphi_N C_N(t)\Delta t$. Therefore, we have

$$I(t+\Delta t) = I(t) - \nu I(t)\Delta t + \psi\varphi_N C_N(t)\Delta t. \tag{3}$$

Proceed to limit for $\Delta t \to 0$ in (3). We obtain the equation $\dot{I} = -\nu I + \psi\varphi_N C_N$ of system (2).

Remark 1. Consider the difference between φ_i of model (1), where φ_i are not constants, and in the proposed model, where φ_i are constants. This difference is not critical, because, in some sense, the multiplier $(V_i - C_i)$ acts as φ_i of model (1). In our model, the multiplier $(V_i - C_i)$ changes its sign while exceeding the economic niche volume. This fact allows to take into account directly the boundedness of the economic growth.

Let us consider the model with the single efficiency level and the intellectual capital.

$$\begin{cases} \dot{C}_1 = \frac{1-\varphi_1}{\lambda_1} C_1(V_1 - C_1) = f_1, \\ \dot{I} = -\nu I + \psi\varphi_1 C_1 = g. \end{cases} \tag{4}$$

Consider the right hand sides of the equations to be equal to zero. We obtain the equilibria: $(0,0)$, (V_1, I^*), where $I^* = \frac{\psi\varphi_1}{\nu}V_1$.

Proposition 1. *The set* $\bar{\mathbb{R}}^2_+ = \{(x,y) : x \geqslant 0, y \geqslant 0\}$ *is invariant.*

Proof. If $C_1 = 0$, then, taking into account (4), we obtain that $\dot{C}_1 = 0$. Therefore, the trajectories do not leave $\bar{\mathbb{R}}^2_+$ through the axis I $(C_1 = 0)$. If $I = 0$, then $\dot{I} = \psi\varphi_1 C_1 \geqslant 0$. Therefore, the trajectories do not leave $\bar{\mathbb{R}}^2_+$ through the axis C_1 $(I = 0)$. Consequently, the trajectories do not leave $\bar{\mathbb{R}}^2_+$, therefore, $\bar{\mathbb{R}}^2_+$ is invariant.

□

Theorem 1. *The equilibrium* $P = (V_1, I^*)$ *is asymptotically stable, the equilibrium* $(0,0)$ *is unstable.*

Proof. The Jacobi matrix is as follows:

$$f' = \begin{pmatrix} -2a_1C_1 + a_1V_1 & 0 \\ \psi\varphi_1 & -\nu \end{pmatrix}. \tag{5}$$

Determine the eigenvalues for the equilibria as the roots of the characteristic polynomials

$$det(f'(V_1, I^*) - \lambda E) = \begin{vmatrix} -a_1V_1 - \lambda & 0 \\ \psi\varphi_1 & -\nu - \lambda \end{vmatrix} = (-a_1V_1 - \lambda)(-\nu - \lambda) = 0. \tag{6}$$

Therefore, $\lambda_1 = -a_1V_1 < 0$, $\lambda_2 = -\nu < 0$. Consequently, the equilibrium $P = (V_1, I^*)$ is asymptotically stable.

$$det(f'(0,0) - \lambda E) = \begin{vmatrix} a_1V_1 - \lambda & 0 \\ \psi\varphi_1 & -\nu - \lambda \end{vmatrix} = (a_1V_1 - \lambda)(-\nu - \lambda) = 0. \tag{7}$$

Here $\lambda_1 = a_1V_1 > 0$. Therefore, $(0,0)$ is unstable.

□

Denote $\mathbb{R}^n_+ = \{(c_1, ..., c_n) : c_i > 0, i = 1, ..., n\}$.

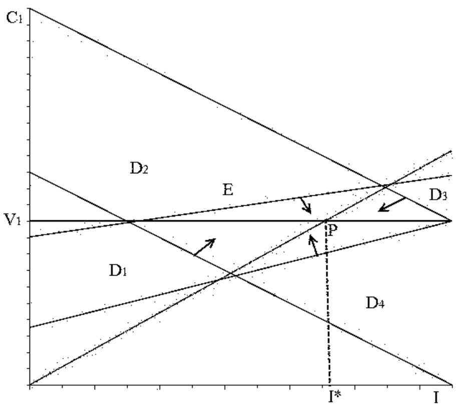
Definition 1. *An equilibrium that belongs to the invariant set* M *is global stable in* M*, if the equilibrium is asymptotically stable in* M *in the sense of Lyapunov for any initial data in* M*.*

Theorem 2. *The equilibrium* $P = (V_1, I^*)$ *is global stable in* $\mathbb{R}^2_+$*.*

Therefore, Theorem 2 shows that the capital volume tends to the economic niche volume, while the intellectual capital volume tends to the value $\frac{\psi\varphi_N}{\nu}V_1$, i.e. is equal to the relation of the scientific investments to the research rate.

Proof. The set $\mathbb{R}^2_+$ is divided by isoclines $C_1 = V_1$ and $I = \tilde{I} = \frac{\psi\varphi_N}{\nu}C_1$ into four domains D_i, $i = 1, .., 4$, as follows (see Figure):

- $D_1 = \{(C_1, I) : C_1 < V_1, I < \tilde{I}\}$,
- $D_2 = \{(C_1, I) : C_1 > V_1, I < \tilde{I}\}$,
- $D_3 = \{(C_1, I) : C_1 > V_1, I > \tilde{I}\}$,
- $D_4 = \{(C_1, I) : C_1 < V_1, I > \tilde{I}\}$.



All trajectories enter any quadrangle E,
and the equilibrium P is global stable

Therefore,
- In D_1: $f_1 > 0, g > 0$,
- In D_2: $f_1 < 0, g > 0$,
- In D_3: $f_1 < 0, g < 0$,
- In D_4: $f_1 > 0, g < 0$.

The equilibrium $P = (V_1, I^*)$ belongs to the domain E with the boundary consisting of the segments of the lines $k_1C_1 + hI = r > 0$, where

$$\begin{cases} k_1 > 0 & C_1 < V_1, \\ k_1 < 0 & C_1 > V_1, \end{cases}$$

$$\begin{cases} h > 0 & I < \tilde{I}, \\ h < 0 & I > \tilde{I}. \end{cases}$$

The normal vectors to these lines are $\overline{n} = (k_1, h)$. Here k_1 and f_1, as well as h and g, have the same signs in each domain. Then

$$(\overline{n}, \overline{f}) = k_1 f_1 + hg > 0. \tag{8}$$

Therefore, the angles between the trajectories and normals are acute and, consequently, the trajectories intersect the lines $k_1C_1 + hI = r$ from the outside to the inside. Hence, all trajectories approach the equilibrium P arbitrary close. Therefore, the equilibrium P is global stable.

□

Let us introduce and analyze the model with an arbitrary number of the efficiency levels:

$$\begin{cases} \dot{C}_1 = \frac{1-\varphi_1}{\lambda_1} C_1(V_1 - C_1) = f_1, \\ \dot{C}_i = \frac{1-\varphi_i}{\lambda_i} C_i(V_i - C_i) + \varphi_{i-1}C_{i-1}, i = 2, .., N = f_i, \\ \dot{I} = -\nu I + \psi\varphi_N C_N = g. \end{cases} \tag{9}$$

Consider the right hand sides of the equations to be equal to zero. We obtain the equilibria: $(0, ..., 0)$, $(0, ..., 0, V_{N-1}, C_N^*, I^*)$, $(0, ..., 0, V_{N-2}, C_{N-1}^*, C_N^*, I^*)$, ..., $(V_1, C_2^*, ..., C_N^*, I^*)$, where $C_i^* = \frac{V_i + \sqrt{{V_i}^2 + \frac{4\varphi_{i-1}}{a_i} V_{i-1}}}{2}$, $a_i = \frac{1-\varphi_i}{\lambda_i}$, $I^* = \frac{\psi\varphi_N}{\nu} {C_N}^*$, $i = 2, .., N$.

Proposition 2. *The set* $\bar{\mathbb{R}}_+^{N+1} = \{(x_1, x_2, ..., x_{N+1}) : x_i \geqslant 0, i = 1, ..., N+1\}$ *is invariant.*

Proof. Consider the boundary hyperplanes of $\bar{\mathbb{R}}_+^{N+1}$. If $C_1 = 0$, then, taking into account (9), we obtain that $\dot{C}_1 = 0$. If $C_i = 0$, then $\dot{C}_i = \varphi_{i-1}C_{i-1} \geqslant 0$. If $I = 0$, then $\dot{I} = \psi\varphi_N C_N \geqslant 0$. It means that the trajectories do not leave $\bar{\mathbb{R}}_+^{N+1}$ through the boundary hyperplanes. Therefore, $\bar{\mathbb{R}}_+^{N+1}$ is invariant. □

Theorem 3. *The equilibrium $P = (V_1, C_2^*, ..., C_N^*, I^*))$ is locally asymptotically stable, the other equilibria are unstable.*

Proof. The Jacobi matrix is as follows:

$$f' = \begin{pmatrix} -2a_1C_1 + a_1V_1 & 0 & \dots & 0 & 0 \\ \varphi_1 & -2a_2C_2 + a_2V_2 & \dots & 0 & 0 \\ \dots & & & & \\ 0 & \dots & \varphi_{n-1} & -2a_NC_N + a_NV_N & 0 \\ 0 & \dots & 0 & \psi\varphi_N & -\nu \end{pmatrix}. \tag{10}$$

Determine the eigenvalues for the equilibria as the roots of the characteristic polynomials.

$$det(f'(V_1, {C_2}^*, ..., {C_N}^*, I^*) - \lambda E) = (-a_1V_1 - \lambda)(-a_2\sqrt{{V_2}^2 + \frac{4\varphi_1}{a_2}V_1} - \lambda)\dots \tag{11}$$
$$\dots(-a_N\sqrt{{V_N}^2 + \frac{4\varphi_{N-1}}{a_N}V_{N-1}} - \lambda)(-\nu - \lambda) = 0.$$

Therefore, $\lambda_1 = -a_1V_1 < 0$, $\lambda_i = -a_i\sqrt{V_i^2 + \frac{4\varphi_{i-1}}{a_i}V_{i-1}} < 0$, $i = 2, ..., N$, $\lambda_{N+1} = -\nu < 0$. Consequently, the equilibrium $P = (V_1, {C_2}^*, ..., {C_N}^*, I^*)$ is locally asymptotically stable. It is clear that if $C_i = 0$, then corresponding $\lambda_i = a_iV_i > 0$. Therefore, the other equilibria are unstable. □

Theorem 4. *The equilibrium $P = (V_1, C_2^*, ..., C_N^*, I^*)$ is global stable in $\mathbb{R}_+^{N+1}$.*

Proof. The set $\mathbb{R}_+^{N+1}$ is divided into 2^{N+1} domains depending on the signs of $\dot{C}_i$, $i = 1, .., N$, and $\dot{I}$. We have $\dot{C}_1 > 0$, if $C_1 < V_1$, $\dot{C}_i > 0$, if $C_i < \tilde{C}_i$, and $\dot{I} > 0$, if $I < \tilde{I}$. The equilibrium $P = (V_1, {C_2}^*, ..., {C_N}^*, I^*)$ belongs to the domain E with the boundary consisting of the hyperplanes $k_1C_1 + \ldots + k_NC_N + hI = r > 0$, where

$$\begin{cases} k_1 > 0 & C_1 < V_1, \\ k_1 < 0 & C_1 > V_1, \end{cases}$$
$$\begin{cases} k_i > 0 & C_i < \tilde{C}_i, \\ k_i < 0 & C_i > \tilde{C}_i, \end{cases}$$
$$\begin{cases} h > 0 & I < \tilde{I}, \\ h < 0 & I > \tilde{I}. \end{cases}$$

The normal vectors to these hyperplanes are $\overline{n} = (k_1, \ldots, k_N, h)$. Here k_i and f_i, as well as h and g, have the same sign in each domain. Therefore,

$$(\overline{n}, \overline{f}) = k_1f_1 + \ldots + k_Nf_N + hg > 0. \tag{12}$$

Consequently, the angles between the trajectories and normals are acute. Hence, the trajectories intersects the hyperplanes $k_1C_1 + \ldots + k_NC_N + hI = r$ from the outside to the inside. Therefore, all trajectories approach the equilibrium P arbitrary close. Consequently, the equilibrium P is global stable. □

Therefore, the capital approaches to the limit value at each efficiency level, and then the system stabilizes. The global stability of the equilibrium means the economic stagnation, which is extremely unfavorable. The further economic growth requires new efficiency level. This can be interpreted as follows: the revolutionary changes are necessary, when the evolutionary changes are impossible. This statement agrees with the concept of the creative destruction proposed by J. Schumpeter.

Theorems 2, 4 allow to predict the behavior of the economic system.

2. Scenario of Emergence of New Highest Efficiency Level

Let us consider the following scenario of the emergence of the new highest efficiency level. Assume that the sum of the economic niche volumes is constant, i.e. $\sum_{i=1}^{N(t)} V_i = V$, where V is a given constant, $N(t)$ is the number of the levels at the moment t, and there is a redistribution of the volumes at the moment when a new level emerges. Herewith, at the new level $N+1$, the initial capital equals to $C_{N+1} = (1-\psi)\varphi_N C_N^*$. The new level emerges when the intellectual capital I achieves the threshold value $I^* - \varepsilon$, where $0 < \varepsilon$ is a sufficiently small value. In what follows we assume that $I(0) < I^* - \varepsilon$, because otherwise the number of levels is $N+1$ or more. Note that we consider the threshold value to be equal to $I^* - \varepsilon$, since the equilibrium $P = (V_1, C_2^*, ..., C_N^*, I^*)$ is global stable. The global stability of P implies that the further capital growth is impossible and, therefore, there is the economic stagnation. If the coordinates of the initial point satisfy the condition $C_N < \tilde{C}_N$, then the trajectory attains the equilibria in an infinite time. Since the intellectual capital I amounts the threshold $I^* - \varepsilon$ in a finite time, then it is reasonable to assume the emergence of the new level at the moment of the threshold $I^* - \varepsilon$ reaching. Otherwise, if the coordinates of the initial point satisfy the condition $C_N > \tilde{C}_N$, then it is reasonable to create the new level when the intellectual capital I achieves the threshold value $I^* - \varepsilon$, since the capital C_N decreases, i.e. the firms production is unprofitable at the N-th level.

Assume that the new level is emerged at the moment $t = t_{em}$. At this moment, there is a momentary transition from $\mathbb{R}_+^{N+1}$ to $\mathbb{R}_+^{N+2}$ such that the point $(C_1(t_{em}), ..., C_N(t_{em}), I^* - \varepsilon)$ moves to the point $(C_1(t_{em}), ..., C_N(t_{em}), C_{N+1}(t_{em}), I(t_{em}))$, where $C_{N+1}(t_{em}) = (1 - \psi)a_N C_N(t_{em})$, $I(t_em) < \frac{\psi\varphi_N}{\nu} C_{N+1}^*$, i.e. the initial value $I(t_{em})$ is sufficiently small. Substitute $C_{N+1}(t_{em})$ in the capital dynamic equation at the $N+1$-th level:

$$\dot{C}_{N+1} = a_{N+1}(1-\psi)\varphi_N C_N(t_{em})(V_{N+1} - (1-\psi)\varphi_N C_N(t_{em})) + \varphi_N C_N(t_{em}). \tag{13}$$

In order to guarantee the functioning of the new $N+1$-th level, it is necessary to assume the capital growth at the initial period, i.e. $\dot{C}_{N+1} > 0$, that gives the necessary condition of the new level emergence:

$$0 < C_N(t_{em}) < \frac{\varphi_N + a_{N+1}(1-\psi)V_{N+1}}{a_{N+1}(1-\psi)^2 \varphi_N^2}. \tag{14}$$

Remark 2. As well, we can propose the new level emergence at the moment when the trajectory intersects the boundary of a neighborhood of the equilibrium $P = (V_1, C_2^*, ..., C_N^*, I^*)$.

Conclusion

In this article, we propose a mathematical model of dynamics of the sector capital distribution over efficiency levels. We develop the approach proposed by V.M. Polterovich and G.M. Khenkin. To this end, we take into account the boundedness of the economic growth. The qualitative analysis of the proposed model is presented. The equilibria of the constructed dynamic model are determined, and their local and global stability are investigated. Moreover, based on the dynamic system with variable dimension, we propose an approach to simulate the new efficiency levels emergence.

Acknowledgements. *This work is supported by Russian Foundation for Basic Research (RFBR), project no. 18-01-00249.*

References

1. Schumpeter J. *Theorie der wirtschaftlichen Entwicklung*. Leipzig, Verlag von Duncker und Humblot, 1911. (in German)
2. Acemoglu D., Cao D. Innovation by Entrants and Incumbents. *HKS Faculty Research Working Paper Series*, 2012, no. 238, pp. 1–13.
3. Christensen C., Raynor M. *The Innovator's Solution*. Harvard, Harvard Business School Press, 2003.
4. Glazev S.Yu. *Teoriya dolgosrochnogo tekhniko-ehkonomicheskogo razvitiya* [The Theory of the Long-Run Technical and Economic Development]. Moscow, VlaDar, 1993. (in Russian)
5. Klimek P., Hausmann R., Thurner S. Empirical Confirmation of Creative Destruction from World Trade Data. *HKS Faculty Research Working Paper Series*, 2012, no. 238, pp. 1–13.
6. Iwai K. Schumpeterian Dynamics. Part 1: An Evolutionary Model of Innovation and Imitation. *Journal of Economic Behavior and Organization*, 1984, vol. 5, no 2, pp. 159–190. DOI: 10.1016/0167-2681(84)90017-9
7. Iwai K. Schumpeterian Dynamics. Part 2: Technological Progress, Firm Growth and "Economic Selection". *Journal of Economic Behavior and Organization*, 1984, vol. 5, no. 3–4, pp. 321–351. DOI: 10.1016/0167-2681(84)90005-2
8. Gelman L.M., Levin M.I., Polterovich V.M., Spivak V.A. [Modelling of the Dynamics of Firms Distribution in Accordance with Efficiency Levels (Ferrous Metal Industry Case)]. *Economics and Mathematical Methods*, 1993, vol. 29, no. 24, pp. 460–469. (in Russian)
9. Polterovich V.M., Khenkin G.M. [An Evolutionary Model with Interaction between Development and Adoption of New Technologies]. *Economics and Mathematical Methods*, 1988, no. 24, pp. 1071–1083. (in Russian)
10. Polterovich V.M. The Theory of Endogenous Economic Growth and Equations of Mathematical Physics. *The Journal of the New Economic Association*, 2017, no. 2 (34), pp. 193–201. DOI: 10.31737/2221-2264-2017-34-2-11
11. Khenkin G.M., Shananin A.A. Mathematical Modelling of the Schumpeterian Dynamics of Innovation. *Mathematical Modelling*, 2014, vol. 26, no. 3, pp. 3–19.
12. Kirillov A.N., Sazonov A.M. [Global Stability in Model of Non-Linear Schumpeterian Dynamics]. *Transactions of KarRC RAS*, 2018, no. 7, pp. 34–40. DOI: 10.17076/mat794

Received November 17, 2018

УДК 517.938+517.925.51+51-77 DOI: 10.14529/mmp190302

ГЛОБАЛЬНАЯ ШУМПЕТЕРОВСКАЯ ДИНАМИКА СО СТРУКТУРНЫМИ ИЗМЕНЕНИЯМИ

***А.Н. Кириллов*[1], *А.М. Сазонов*[1]**
[1] Институт прикладных математических исследований, Федеральный исследовательский центр «Карельский научный центр РАН», г. Петрозаводск, Российская Федерация

В статье предлагается подход к моделированию шумпетеровской динамики экономической системы, описывающей возникновение и распространение новых технологий. Разработана математическая модель динамики распределения капитала отрасли по уровням эффективности на основе системы нелинейных дифференциальных уравнений. С целью учета ограничений экономического роста, вследствие ограниченности рынков сбыта и ресурсной базы, вводится понятие емкости экономической ниши. Предложен сценарий появления нового высшего уровня эффективности. Для моделирования процесса появления нового высшего уровня эффективности введено понятие интеллектуального капитала. Согласно предложенному сценарию, новый уровень появляется при достижении интеллектуальным капиталом порогового значения. При этом изменяется размерность динамической системы. Сформулировано необходимое условие функционирования нового уровня. Определено инвариантное множество динамической системы. Исследуется локальная устойчивость состояний равновесия динамической системы, описывающей распределение капитала. С помощью геометрического метода устанавливается глобальная устойчивость одного из равновесий. Предложенные модели позволят оценивать и прогнозировать динамику технологических уровней развития предприятий отрасли экономики.

Ключевые слова: динамические системы; шумпетеровская динамика; устойчивость.

Литература

1. Schumpeter, J. Theorie der wirtschaftlichen Entwicklung / J. Schumpeter. – Leipzig: Verlag von Duncker und Humblot, 1911.
2. Acemoglu, D. Innovation by Entrants and Incumbents / D. Acemoglu, D. Cao // HKS Faculty Research Working Paper Series. – 2012. – № 238. – P. 1–13.
3. Christensen, C. The Innovator's Solution / C. Christensen, M. Raynor. – Harvard: Harvard Business School Press, 2003.
4. Глазьев, С.Ю. Теория долгосрочного технико-экономического развития / С.Ю. Глазьев. – М.: ВлаДар, 1993.
5. Klimek, P. Empirical Confirmation of Creative Destruction from World Trade Data / P. Klimek, R. Hausmann, S. Thurner // HKS Faculty Research Working Paper Series. – 2012. – № 238. – P. 1–13.
6. Iwai, K. Schumpeterian Dynamics. Part 1: An Evolutionary Model of Innovation and Imitation / K. Iwai // Journal of Economic Behavior and Organization. – 1984. – V. 5, № 2. – P. 159–190.
7. Iwai, K. Schumpeterian Dynamics. Part 2: Technological Progress, Firm Growth and ≪Economic Selection≫ / K. Iwai // Journal of Economic Behavior and Organization. – 1984. – V. 5, № 3–4. – P. 321–351.

8. Гельман, Л.М. Моделирование динамики распределения предприятий отрасли по уровням эффективности (на примере черной металлургии) / Л.М. Гельман, М.И. Левин, В.М. Полтерович, В.А. Спивак // Экономика и математические методы. – 1993. – Т. 29, № 3. – С. 460–469.
9. Полтерович, В.М. Эволюционная модель взаимодействия процессов создания и заимствования технологий / В.М. Полтерович, Г.М. Хенкин // Экономика и математические методы. – 1988. – № 24. – С. 1071–1083.
10. Полтерович, В.М. Теория эндогенного экономического роста и уравнения математической физики / В.М. Полтерович // Журнал Новой экономической ассоциации. – 2017. – № 2. – С. 193–201.
11. Хенкин, Г.М. Математическое моделирование шумпетеровской инновационной динамики / Г.М. Хенкин, А.А. Шананин // Математическое моделирование. – 2014. – Т. 26, № 8. – С. 3–19.
12. Кириллов, А.Н. Глобальная устойчивость в модели нелинейной шумпетеровской динамики / А.Н. Кириллов, А.М. Сазонов // Труды КарНЦ РАН. – 2018. – № 7. – С. 34–40.

Александр Николаевич Кириллов, доктор физико-математических наук, ведущий научный сотрудник, Институт прикладных математических исследований, Федеральный исследовательский центр «Карельский научный центр РАН» (г. Петрозаводск, Российская Федерация), krllv1812@yandex.ru.

Александр Михайлович Сазонов, аспирант, Институт прикладных математических исследований, Федеральный исследовательский центр «Карельский научный центр РАН» (г. Петрозаводск, Российская Федерация), sazonov@cs.karelia.ru.

Поступила в редакцию 17 ноября 2018 г.