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CHOOSING AVERAGE VALUES WHEN DETERMINING CHARACTERISTICS OF THE UNSTEADY BOILING OF LIQUID

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This paper presents an analysis of the issues associated with constructing mathematical models for processes of intense phase transformations and, in particular, focuses on the aspect of using closing relations of empirical origin. The main trend in the implementation of modern numerical algorithms for practical problems is aimed at improving the accuracy of calculation results. The latter is usually achieved by refining a certain set of coefficients in mathematical models. These refinements are carried out both on the basis of the modernization of existing approaches, and with the involvement of new empirical information obtained for a limited number of regime conditions. Predictive models for describing the dynamics of phase transformations, as one of the most difficult in the mathematical formulations, refer to a particularly striking manifestation of the problem under study. In this research, we discuss the existing and widely used experimental work devoted to the extraction of primary information about the dynamics of vapor bubbles on the surface of metal heaters. Their example reveals the presence of a simplified approach in the existing development methodology, and shows a way to determine the correct generalization of empirical information that has a pseudo-stochastic nature.

Keywords: mathematical models; nucleate boiling; averaging.

Introduction

The widespread use of the boiling process in power plants is stipulated by the possibility of transferring a large amount of heat from the specific surface area at constant temperature of the materials involved in this process. In a wide range of research works that address both the experimental study of the boiling process [1–5] and aspects of its modelling [6–9], only a small part of research is devoted to the formulation of the problem with unsteady heat transfer [10, 11]. This is caused by complex implementation of experimental work for an unsteady problem statement, imperfection of modern computational algorithms, the lack of understanding of the mechanics of the processes under consideration associated with interphase transformations, and the difficulty of generalizing boiling characteristics. Each of these problems is difficult to solve even individually, but it is also worth noting that they influence each other. Incompleteness of empirical information causes imperfection of mathematical models, which, in turn, reduce the quality of the information obtained during the experiment. In most cases, under unsteady conditions, many characteristics with the necessary spatial and temporal discretization (temperatures, geometric dimensions of multiphase structures, etc.) can be determined only numerically. The fact that there are some successful implementations of optical methods for these problems does not cancel specific requirements for their implementation. For example, we need to use rather thin metal conductors and employ optical methods to obtain the detailed information about the behavior of temperature and heat fluxes. At the same time, limitations of tracer diagnostic methods are quite obvious when we deal with a large amount of the vapor phase in the studied devices and vessels.

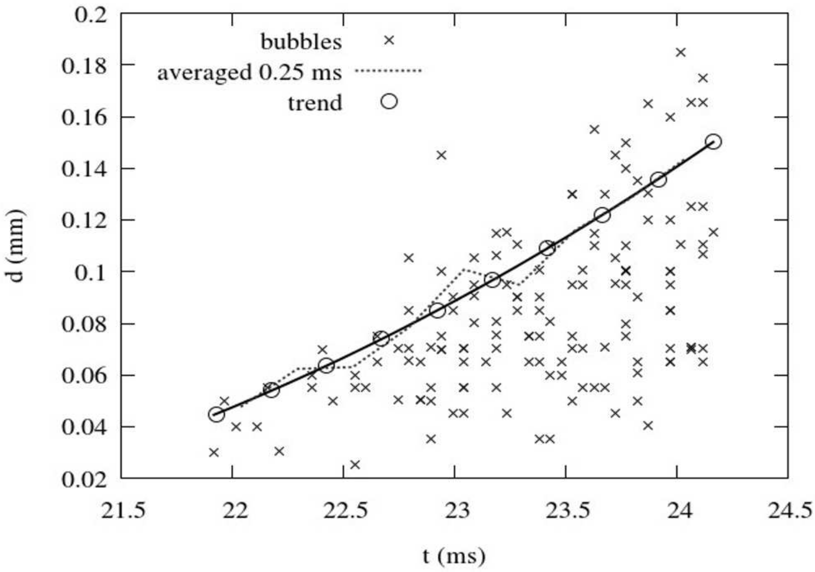
1. Nucleate Boiling Characteristics

The statistical nature of the quantities characterizing boiling can be attributed to both the scale of the initial vapor nucleus and the complex picture of the dynamics in the near-wall layer of the liquid. Some studies show that the surface temperature may oscillate within a single nucleation event by more than 5–10 K, and microconvection in the vicinity of bubbles is responsible for heat transfer up to 25% of the total heat flux. The problem of insufficient empirical data even for cases with stationary bubble boiling is well known. When evaluating the characteristics of nucleate boiling, it is recommended to carry out measurements of at least 300 units within the framework of a single observation. As can be seen from Table, such estimates cast doubt on the completeness of experimental work in this area of research. At the same time, since such parameters of nucleate boiling as the geometric dimensions of vapor structures (bubbles) are stochastic, the derived parameters that depend on them are also stochastic (they include local heat fluxes, thickness of the thermal layer of the liquid, etc.). However, for practical problems, averaged characteristics are of the greatest importance; therefore, researchers create various empirical estimates of the integral heat transfer characteristics and boiling properties.

Table

Study	Murshed et al. [12]	Prodanovic [13]	Situ [14]	Klausner [15]	Klausner [16]
Measurement conditions	D_d, f_d 3	D_d 54	D_d, f_d 92	D_d 35	D_d 19
Orientation	Vertical	Vertical	Vertical	Horizontal	Vertical
Fluid	R134a	Water	Water	R113	FC87
Heated surface	Stainless steel	Stainless Steel	Stainless Steel	Nichrome	Nichrome
Geometry	Rectangular	Annular	Annular	Square	Square
Hydr. Diameter, [mm]	5,56	9,3	19	25	12,7
Pressure, [kPa]	690, 758, 827	105–300	150, 300, 450	131–212	142–155
Heat flux, [kW/m^2]	130	100–1200	100–492	11–26	1,32–14,6
Mass flux, [kg/m^2 s]	1206	79–790	235–986	113–287	192–666
Subcooling, [C]	4, 7, 9,5	10–30	5–40	1–19	1,96–4,91
Measurements per data point	No data	100	100	200	~ 57

Thus, since it is necessary to construct an array of experimental data on the unsteady boiling processes, the choice of the correct way of generalizing the data becomes even more important. Figure shows the scatter of bubble sizes in experiments with transient boiling on the surface of a stainless steel heater. During brief moments of time, dozens of milliseconds, the maximum size of the bubbles changes considerably, while the minimum size does not change throughout the entire process. This is caused by the chaotic distribution of roughness on technical surfaces, where the size of the depressions can vary within wide limits, which makes a contrast with most studies that deal with smooth, almost ideal, surfaces. As a result, an increase in the surface temperature leads to the activation of an increasing number of potential vaporization centers, thus giving rise to a spectrum of possible bubble sizes on the surface.



Dynamics of the biggest bubbles diameters on the surface of a stainless steel tubular heater, $T_b = 30$ C. The wall heating rate is 6 K/ms

2. Parameters Averaging

The choice of the characteristic bubble size corresponding to the conditions of a particular case under study requires an even greater availability of an extensive initial database. For some values of experimental data, the scatter of uncertainty in individual measurements leads to more than 50% of the uncertainty of the average value. Such empirical information is basically useless for studying boiling processes and may only help yield some qualitative conclusions with any analytical generalizations that could be applied to quantitative predictions. The other side of this problem is the way the average value is chosen. In mathematics, the following methods are traditionally used: determining the arithmetic mean (with or without neglecting extreme values), calculating the median value, determining the geometric mean, finding the weighted average and the harmonic mean. Let us consider some of these approaches as applied to the problems of thermal physics.

1. Choose the maximum value of the diameter from the possible values. This approach can be based on the idea of realizing optimal conditions for selective centers of vaporization that have the greatest impact on the process under study.
2. Choose the arithmetic mean with a given quantization in time of the process for transient states or without the need for temporal quantization. This is one of the most widespread approaches, the validity of which is mostly justified by the habit to

generalize all data assuming that the contribution of each individual act of nucleation to the overall boiling process is equal.

3. Integrate the characteristics of a material (liquid, gas, or metal) along the length of long heat exchangers. The most common example of this is the temperature in lumped mathematical models used to describe equipment dynamics. In a mathematical sense, this technique is a truncation of the original system of differential equations, describing the dynamics of momentum balance, mass, and energy, by integrating over the spatial coordinate. So, the energy equation for calculating the dynamics of a recuperative heat exchanger takes the form:

$$\overline{\rho}_l V_l \frac{d\bar{i}_l}{d\tau} + D_l \left(i_{l.2} - i_{l.1}\right) = \alpha_l F_l \left(\overline{\theta} - \overline{T}_l\right), \tag{1}$$

where $\overline{\rho}_l$ is the water density at the temperature $\overline{T}_l$ averaged over the heat exchanger length, V_l is the inner volume of the heat exchanger taken by water, $\bar{i}_l$ is the average water enthalpy, D_l is the water rate, $i_{l.2}$ is the water enthalpy at the heat exchanger outlet, $i_{l.1}$ is the water enthalpy at the heat exchanger inlet, α_l is the heat transfer coefficient, F_l is the heat exchange area, $\overline{\theta}$ is the average metal temperature in the heat exchanger. Determination of the correct method of averaging over the length of heat exchangers is the subject of several studies. In a general case, an additional coefficient k is introduced, and the distribution of parameters takes the form:

$$\bar{i}_l = k \cdot i_{l.2} - (1-k) i_{l.1}. \tag{2}$$

In practical implementations, $\bar{i}_l$ is determined by numerical solution of (1), and the enthalpy at the outlet $i_{l.2}$ is found from (2). The use of any constant value of k inevitably leads to significant errors, since the derivative of the temperature $T(x, \tau)$ with respect to time cannot be equal to 0. Moreover, since the solution to (1) does not describe the transport delay, the perturbation of enthalpy at the heat exchanger inlet $\Delta i_{l.1}$ is described by a mathematical model with an incorrect intermediate result $\Delta i_{l.2} \approx -\Delta i_{l.1}$ for small time steps $\Delta\tau$.

Generally, truncating differential equations by integration along the heater length inevitably leads to the loss of information about the form of the dependence $T(x)$,τ), which is an essential part of the dynamic properties of the object. This technique might be replaced by transferring the integration into numerical modelling algorithms, while preserving the information about the state of the modeled object for a given period of time equal to the time constant.

4. Find the parameter weighted average value. Any value, specific for the process under study, can be such a parameter. When describing the closing relations for nucleate boiling, the most important parameter is the value of the heat flux, since the main problem is to determine the heat transfer between a solid surface and a liquid. During the formation and collapse of bubbles, the scales can be related both to the bubble area and the bubble volume. Thus, it is necessary to average the bubble diameters and implement the result into calculation. The choice of the characteristic value (the bubble's area or volume) should be determined by the type of mathematical dependence of the heat flux component on the geometric characteristic.

To analyze the thermal picture near the heater under conditions of unsteady heat release accompanied by boiling of a moving flow of the water subcooled to the saturation temperature, a numerical modelling was performed earlier in [17] using COMSOL. It was shown that different geometric characteristics are used to determine the various components of the heat flux in the near-wall liquid layer (thermal conductivity through the superheated layer, thermal conductivity and evaporation of the microlayer, condensation of the evaporated volume).

$$q = q_{ei} + q_{eml} + q_{cml} + q_{cs}. \tag{3}$$

Here the heat balance components are defined as:

$$q_{ei} = h_{lg} \rho_g \frac{\pi D_m^3}{6} f N_a, \tag{4}$$

$$q_{eml} = h_{lg} \rho_l \delta_{ml} \frac{\pi D_{ml}^2}{6} f N_a, \tag{5}$$

$$q_{cml} = \frac{N_a \pi k_l \left(T_w - T_s\right) D_{ml}^2}{4 \delta_{ml}}, \tag{6}$$

$$q_{cs} = N_a \int_{\delta_{ml}}^{\delta_s} k_l \left(T_w - T_s\right) \pi \left(\frac{D_m}{r - r_w} - 2 \right) dr, \tag{7}$$

where D_{ml} is the diameter of the microlayer proportional to the maximum bubble size D_{ml}, while T_w is the wall temperature, T_s is the saturation temperature, N_a is the density of vaporization centers, f is the bubble formation frequency, h_{lg} is the latent vaporization heat, δ_{ml} is the microlayer thickness, k_l is the thermal conductivity of the liquid, δ_{ml} is the thickness of the superheated layer of the liquid, ρ is the medium density.

Analyzing equations (3) – (7), it is easy to see that the heat transfer through the superheated liquid layer is proportional to the bubble diameter, while the flow components q_{cml} and q_{eml} associated with the microlayer are proportional to the surface area under the bubble, and the heat of the condensed bubble q_{ei} is, of course, proportional to its volume. Hence, we can conclude that, for each of the components of the heat flow in particular, and for each determined parameter in the mathematical model in general, it is necessary to use the appropriate measure when finding weighted averages. An incorrect choice of a method for averaging the empirically obtained bubble sizes for the subsequent use of generalized values leads to an additional error. It can be readily shown that if the volume-weighted average diameter is replaced by a simple arithmetic mean, the error introduced by this into the heat transfer calculations is about 15%.

Conclusion

The presented analysis of the problems in construction of mathematical models for liquid boiling addressed the influence of correct generalization of empirical data used to construct closing relations. As an example of the fundamental importance of choosing a method for generalizing statistically inhomogeneous measurement results, we considered such parameters as the geometric dimensions of vapor structures (bubbles) and their temporal characteristics (waiting and growth times, as nucleation frequencies). The generated error caused by incorrect handling of empirical data can be very significant, far exceeding the error of direct measurements. Another specific issue for further study is the indicators of thermophysical data variation, such as the variation range, dispersion and distribution law. Some results on the analysis of statistical heterogeneity of data are presented in a number of works.

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О ВЫБОРЕ СРЕДНИХ ВЕЛИЧИН ПРИ ОПРЕДЕЛЕНИИ ХАРАКТЕРИСТИК НЕСТАЦИОНАРНОГО ПОВЕРХНОСТНОГО КИПЕНИЯ ЖИДКОСТИ

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В работе представлен анализ проблем построения математических моделей процессов интенсивных фазовых превращений, а именно аспекту использования замыкающих соотношений эмпирического происхождения. Основной тренд при реализации современных численных алгоритмов для практических задач направлен на улучшение точности результатов расчета, достигаемое обычно за счет уточнения некоторого набора коэффициентов в математических моделях. Эти уточнения осуществляются как на основе модернизации существующих подходов, так и с привлечением новых эмпирических сведений, получаемых для ограниченного числа режимных условий. Предсказательные модели для описания динамики фазовых превращений, как одни из наиболее сложных в математической постановке задач, относятся к особо яркому проявлению описываемой проблемы. Рассмотрены существующие и широко применяемые экспериментальных работ, посвященные извлечению первичных сведений о динамике паровых пузырей на поверхности металлических нагревателей. На их примере показано наличие упрощенного подхода в существующей методологии разработки, и показан способ определения корректного обобщения эмпирических сведений, обладающих псевдо-стохастическим характером.

Ключевые слова: математические модели; пузырьковое кипение; усреднение.

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