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ITERATIVE LEARNING CONTROL ON NONLINEAR STOCHASTIC NETWORKED SYSTEMS WITH NON-DIFFERENTIABLE DYNAMICS

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In the design of iterative learning control (ILC) algorithm for stochastic nonlinear networked systems, the underlying assumption is differentiability of the system dynamics. In many cases, in reality, stochastic nonlinear networked systems have non-differentiable dynamics, but their dynamics functions after discretization by using conventional methods have global Lipschits' continuous (GLC) condition. In this paper, we apply an ILC algorithm for stochastic nonlinear networked systems that have the GLC condition. We demonstrate that to design the ILC algorithm, differentiability of the system dynamics is not necessary, and the GLC condition is sufficient for designing the ILC algorithm for stochastic nonlinear networked systems with non-differentiable dynamics. We investigate the analysis of convergence and the tracking performance of the proposed update law for stochastic nonlinear networked systems with GLC condition. We show that there exists no limited condition for the stochastic data dropout probabilities in the convergence investigation of the input error. Then, the results are reviewed and confirmed with a numerical example.

Keywords: iterative learning control; stochastic nonlinear networked system; non-differentiable; global Lipschits continuous (GLC); data dropout.

Introduction

The use of human learning and experience in doing tasks and success in them is an incentive to develop a control method called iterative learning control. People learn by practicing and repeating a task to perfect it. Due to the nature of ILC, ILC can be used to improve performance in systems that perform repetitive tasks in a limited amount of time. ILC is a control method that is designed for amending tracking control performance when a system is performing a repeating task. The ILC algorithm deals with the reference tracking control problem, which repeats the desired trajectory in a limited time called trial length. ILC is applied to many industrial applications, such as injection molding, robotics, rolling mills, and chemical patch processes, etc. The fundamental idea is to use the tracking error to update the control input signal in the current trial of ILC, and the aim is to attain better tracking performance from trial to trial. The main idea of ILC goes back to the article [1], which is one of the sources of this topic. ILC is extensively studied and researched in theory and is used in practice as well. The paper [2] is a survey that provides an overview of ILC research, and the paper [3] categorizes almost all ILC algorithms proposed between 1998 and 2004 from various aspects, such as mathematical formula, application type, and system type. Stochastic iterative learning control (SILC) is given by iterative learning control concerning systems with stochastic signals such as random data dropout, measurement noise, and system noise. Previously, there are many kinds of research on various issues of ILC, such as robustness, stability, update law design, frequency analysis, and applied research. The papers [2–7] are survey papers in this area of research. The paper [8] presents an ILC algorithm with gain adaptation for discrete-time stochastic systems. The algorithm is based on Kesten's accelerated stochastic approximation algorithm.

With the advancement of Internet services, with the help of network techniques and communication, control systems can be networked to have robustness, high performance, flexibility, facility, and low cost. Control systems with these features organize network control systems (NCSs). In NCSs, the network locates between the controller and the system. The goal is efficient performance in case of lost data. Because of the random lost data, the common control techniques and classical approximation are not used in NCSs. There are several articles on the employment of the ILC algorithm in NCSs according to the data conditions, the convergence analysis, and the design of the compensation structure. The work [9] is a survey concerning ILC with incomplete information and relevant control systems. In [10], a networked iterative learning control (NILC) is designed for a class of uncertain discrete-time nonlinear systems with random packet dropout and communication delay, where the input-output coupling parameter (IOCP) is assumed to be unknown. The work [11] presents a novel networked iterative learning control scheme with an adjustment factor for a class of discrete-time uncertain nonlinear systems with stochastic input and output packet dropout modelled as 0-1 Bernoulli-type random variable.

In the research field of ILC, for nonlinear systems, the following two categories in terms of system nonlinearities are considered: system dynamic with the GLC condition and system dynamic with the local Lipschitz continuous (LLC) functions. In [7], the author designed linear ILC for GLC nonlinear systems and nonlinear ILC for LLC nonlinear systems.

In practice, many industrial plants have nonlinear systems such that, in many cases, ILC design requires that the dynamics of the nonlinear systems is differentiable. However, there exist nonlinear systems, which dynamics are not differentiable. For example, the work [12] explains that the Hamilton–Jacobi equation is a first-order nonlinear partial differential equation for the value function, which is differentiable. However, there exist generally cases such that the value function is not differentiable. In practice, some systems have non-differentiable dynamics. For example, the dynamics of impulsive systems and backlash in gears may be non-differentiable. Therefore, this motivated us to design an ILC algorithm to improve the performance of such systems with non-differentiable dynamics. In this paper, it is assumed that systems with non-differentiable dynamics have data dropout and are networked.

We can apply a more relaxed condition for stochastic nonlinear networked systems such that their dynamic is not differentiable. In this paper, we design an ILC update law for stochastic nonlinear networked systems such that the system dynamics are not differentiable, but they have the GLC condition after discretization by using conventional methods. We determine that to design the ILC update law, differentiability of the system dynamics is not necessary, while the GLC condition is sufficient to generate the ILC update law for stochastic nonlinear networked systems with non-differentiable dynamics. We analyze convergence and the tracking performance of the recommended algorithm for stochastic nonlinear networked systems that have GLC condition.

The aim of this paper is to show that the non-differentiability of the system dynamics is not an obstacle for the design of the ILC algorithm and is possible by considering a more relaxed condition, namely, the GLC condition.

In the convergence analysis of the ILC algorithm, although the dynamics of the stochastic nonlinear networked system are not differentiable, there is no problem in proving the convergence theorem, and the GLC condition is sufficient to prove the convergence of the ILC algorithm.

Notations. We denote the real number field by $\mathbb{R}$. "P" shows the probability of an event. "E" indicates mathematical expectation. Superscript "T" indicates the transpose of a vector or matrix. $|.|$ denotes absolute value. "i.o." indicates "infinitely often", "a.s." shows "almost surely" and "w.p.1." denotes "with probability one". "i.i.d." stands for "independent and identically distributed".

The paper is organized as follows. In Section 2, we formulate the problem statement and present an ILC update law for stochastic nonlinear networked systems that have the GLC condition. Section 3 investigates the convergence analysis of the proposed algorithm. In Section 4, we consider a numerical example, and, in Section 5, we draw a conclusion.

1. Problem Statement

Consider a continuous-time stochastic nonlinear networked system with non-differentiable dynamics. After discretization by conventional methods such as Euler method, the system is reduced to the following discrete-time nonlinear networked system with stochastic measurement noise:

$$\begin{aligned} x_k(t+1) &= f(t, x_k(t)) + g(t, x_k(t))u_k(t), \\ y_k(t) &= C(t)x_k(t) + \zeta_k(t), \end{aligned} \tag{1}$$

where $k = 1, 2, ...$ indicates the iteration index, $t = 0, 1, ..., N$ is the time index, and the given positive integer N is the iteration length. $u_k(t) \in \mathbb{R}$, $y_k(t) \in \mathbb{R}$, and $x_k(t) \in \mathbb{R}^n$ are the input vector, the output vector, and the state vector, respectively. The stochastic variable $\zeta_k(t)$ indicates measurement noise. Nonlinear functions $f(t, x_k(t))$, $g(t, x_k(t))$, and time-varying vector $C(t)$ represent unknown information on the system.

In ILC networked control systems, the networks are applied to communicate between the iterative learning controller and the operational system. We use the Bernoulli random variables to denote the lost data in this paper, similar to some literature about NCSs. We use $\alpha_k(t)$ with the Bernoulli distribution for modelling the transmission of $y_k(t)$. Therefore, if $y_k(t)$ is not successfully transmitted, then $\alpha_k(t) = 0$, and if $y_k(t)$ is successfully transmitted, then $\alpha_k(t) = 1$. We assume that the probability of successfully output transmission is $0 < r < 1$. Therefore, $\mathrm{P}(\alpha_k(t) = 0) = 1 - r$, $\forall k$, t and $\mathrm{P}(\alpha_k(t) = 1) = r$. Hence, we conclude that $\mathrm{E}\alpha_k(t) = r$.

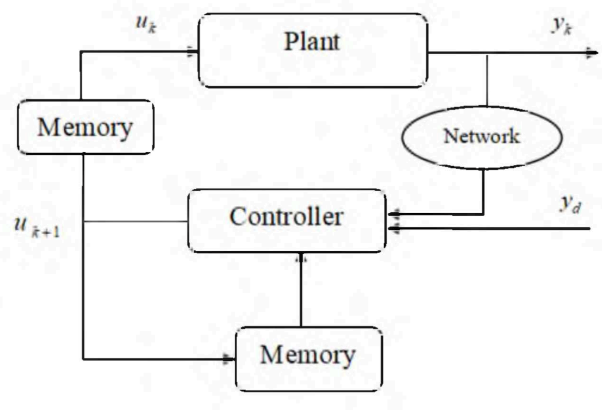

Fig. 1. A control system with a network at the measurement side

Some of the assumptions are as follows.

A1. There exists a unique $u_d(t)$ for generating the desired output $y_d(t)$ with the initial state $x_d(0)$, where $y_d(0) = C(0)x_d(0)$ is such that

$$\begin{aligned} x_d(t+1) &= f(t, x_d(t)) + g(t, x_d(t))u_d(t), \\ y_d(t) &= C(t)x_d(t). \end{aligned} \tag{2}$$

A2. $\forall t$ the i.i.d measurement noise sequence $\zeta_k(t), k = 0, 1, ...$ has $\mathrm{E}\zeta_k(t) = 0$, $\sup_k \mathrm{E}|\zeta_k(t)|^2 < \infty$, and $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \zeta_k(t)\ (\zeta_k(t))^T = \zeta_t$ a.s. such that ζ_t is an unknown matrix.

Remark 1. The condition utilized to the noise measurement is based on the iteration axis rather than the time axis, and, since the process is performed repeatedly and independently, therefore, there are no rigorous conditions.

A3. $\forall\, t = 0, 1, ..., N$, nonlinear functions $f(t, x_k(t))$ and $g(t, x_k(t))$ have the GLC condition, that is, $\forall\, x_1,\, x_2 \in \mathbb{R}^n$, $|f(t, x_1) - f(t, x_2)| \le l_f\, |x_1 - x_2|$ and $\forall\, x_1,\, x_2 \in \mathbb{R}^n$, $|g(t, x_1) - g(t, x_2)| \le l_g\, |x_1 - x_2|$, where $l_f > 0$ and $l_g > 0$ are the Lipschitz constants.

Remark 2. This is the GLC condition, which we use to design the ILC algorithm and analyze its convergence.

A4. It is supposed that the sign of $C(t+1)g(t, x_k(t))$ does not change during the learning process, and its unknown value is nonzero. Therefore, it is assumed that $C(t+1)g(t, x_k(t)) > 0$.

Remark 3. $C(t+1)g(t, x_k(t))$ indicates the control direction. Assumption A4 is necessary because otherwise a plan must be design to find the direction of right control, which complicates the controller.

A5. The measurement noise sequence and the initial state sequence are mutually independent. Furthermore, the i.i.d initial state sequence is resetting asymptotically in the sense that $x_k(0) \to x_d(0)$, w.p.1, when $k \to \infty$.

Remark 4. This technique is necessary to realize the asymptotically re-initialization condition. Remarkably, the classical identical initial condition is a particular case of A5.

In Figure 1, the system output of the current iteration transfers via the network to the ILC controller. In this paper, we consider just data dropouts on the measurement side.

Remark 5. If we consider lost data on both actuator and measurement sides, a more comprehensive investigation is required because we need to consider the asynchronous update between the control signal created by the learning controller and the control signal fed to the plant. Since this is beyond the scope of this paper, for example, refer to [13] for more study.

A mechanism is needed to ensure the convergence of the input error to zero and to overcome the effect of random noise on random systems. Therefore, to eliminate the effect of measurement noise, ensure input convergence and prevent unstable conditions, we consider the following decreasing sequence ρ_k for the proposed update algorithm:

$$\rho_k > 0,\ \rho_k \to 0,\ \sum_{k=1}^{\infty} \rho_k = \infty,\ \sum_{k=1}^{\infty} \rho_k^2 < \infty,\ \forall k = 1,\, 2,\, ... \tag{3}$$

In this paper, we design the ILC algorithm for stochastic nonlinear systems with non-differentiable dynamics with GLC condition for updating and generating inputs due to minimizing $\limsup\limits_{n\to\infty}\frac{1}{n}\sum\limits_{k=1}^{n}|y_d(t)-y_k(t)|^2, \forall t=0,1,...,N$ under the lost data conditions. Due to the unpredictable measurement noise, this article deals with the direct convergence of system inputs to the desired input.

Indeed, we can not use conventional update control laws for networked systems. In this paper, the following ILC update law is proposed under stochastic measurement noises for stochastic nonlinear systems that have GLC condition:

$$u_{k+1}(t) = u_k(t) + \rho_k E_k(t+1), \tag{4}$$

where

$$E_k(t) = \begin{cases} e_k(t), & if\ \alpha_k(t) = 1, \\ 0, & if\ \alpha_k(t) = 0, \end{cases} \tag{5}$$

$e_k(t) = y_d(t) - y_k(t)$ is the tracking error.

We present the convergence analysis of ILC update law (4), (5) as well as its performance evaluation for stochastic nonlinear networked systems with non-differentiable dynamics that after discretization have GLC condition.

2. Convergence Analysis

Let us show the convergence analysis of the recommended law (4), (5). For brevity, we use the symbols $f_k(t) = f(t, x_k(t))$, $f_d(t) = f(t, x_d(t))$, $g_k(t) = g(t, x_k(t))$, $g_d(t) = g(t, x_d(t))$, $\delta f_k(t) = f_d(t) - f_k(t)$, and $\delta g_k(t) = g_d(t) - g_k(t)$. The status error is defined with $\delta x_k(t) = x_d(t) - x_k(t)$. We use the following statement to prove the convergence of the algorithm (4), (5).

Lemma 1. *For system (1), assumptions A1–A5 are considered. If* $\lim\limits_{k\to\infty}\delta u_k(m) = 0$, $m = 0, 1, ..., t$, *then we have* $|\delta f_k(t+1)| \xrightarrow[k\to\infty]{} 0$, $|\delta g_k(t+1)| \xrightarrow[k\to\infty]{} 0$, *and* $|\delta x_k(t+1)| \xrightarrow[k\to\infty]{} 0$, *w.p.1, at the time* $t+1$.

Proof. Taking into account (1) and (2), we conclude that

$$\delta x_k(t+1) = \delta f_k(t) + g_k(t)\delta u_k(t) + \delta g_k(t) u_d(t). \tag{6}$$

Let us prove by mathematical induction.

Initial step. Let $t = 0$, we have

$$\delta x_k(1) = \delta f_k(0) + g_k(0)\delta u_k(0) + \delta g_k(0) u_d(0) \tag{7}$$

Concerning A5, $\delta x_k(0) \xrightarrow[k\to\infty]{} 0$, take into account A3 $\delta f_k(0) \xrightarrow[k\to\infty]{} 0$ and $\delta g_k(0) \xrightarrow[k\to\infty]{} 0$. Since $|g_k(0)| \le |g_d(0)| + |\delta g_k(0)|$, we conclude that $g_k(0)$ is bounded. Therefore, considering assumptions of Lemma 1, $\delta u_k(0) \xrightarrow[k\to\infty]{} 0$, we have $g_k(0)\delta u_k(0) \xrightarrow[k\to\infty]{} 0$. Also, $u_d(0)$ is the initial desired input vector, therefore, $u_d(0)$ is bounded. Hence, we conclude that $\delta g_k(0) u_d(0) \xrightarrow[k\to\infty]{} 0$. Therefore, from (7), we conclude that $\delta x_k(1) \xrightarrow[k\to\infty]{} 0$. Now, concerning A3, we have $\delta f_k(1) \xrightarrow[k\to\infty]{} 0$ and $\delta g_k(1) \xrightarrow[k\to\infty]{} 0$.

Inductive step. Suppose that the results of Lemma 1 hold for $i = 0, 1, ..., t$. We show that conclusions are correct for $t + 1$. The method of proof is the same as the initial step of induction. Therefore, it is proven that $\delta f_k(t+1) \xrightarrow[k\to\infty]{} 0$, $\delta g_k(t+1) \xrightarrow[k\to\infty]{} 0$ and $\delta x_k(t+1) \xrightarrow[k\to\infty]{} 0$. □

In this regard, for convergence investigation, the following statement is proved.

Theorem 1. *Suppose that assumptions A1-A5 hold for the stochastic nonlinear networked system (1). If* $|1 - \rho_k C(t+1) g_k(t)| < 1$*, then for* $u_k(t)$ *updated by ILC update law (4), we conclude that* $u_k(t) \to u_d(t)$ *w.p.1, for all* t *when* $k \to \infty$.

Proof. To prove convergence it is necessary to show that $\delta u_k(t) = u_d(t) - u_k(t) \to 0$ for all $t = 0, 1, ..., N$ when $k \to \infty$.

Considering (1) and (2), according to (4) and (5), we have

$$\begin{gathered}\delta u_{k+1}(t) = \delta u_k(t) - \rho_k E_k(t+1) = \delta u_k(t) - \rho_k \alpha_k(t+1) e_k(t+1) = \\ = \delta u_k(t) - \rho_k \alpha_k(t+1) C(t+1)(x_d(t+1) - x_k(t+1)) + \\ + \rho_k \alpha_k(t+1) \zeta_k(t+1).\end{gathered} \tag{8}$$

Therefore, again considering (1) and (2), we conclude that

$$\begin{gathered}\delta u_{k+1}(t) = \delta u_k(t) - \rho_k C(t+1) g_k(t) \delta u_k(t) + \rho_k C(t+1) g_k(t) \delta u_k(t) - \\ - \rho_k \alpha_k(t+1) C(t+1) \delta f_k(t) - \rho_k \alpha_k(t+1) C(t+1) \delta g_k(t) u_d(t) - \\ - \rho_k \alpha_k(t+1) C(t+1) g_k(t) \delta u_k(t) - \rho_k \alpha_k(t+1) \zeta_k(t+1).\end{gathered} \tag{9}$$

Hence, we have

$$\begin{gathered}\delta u_{k+1}(t) \; = [1 - \rho_k C(t+1) g_k(t)] \delta u_k(t) + \rho_k C(t+1) g_k(t) \delta u_k(t) - \\ - \rho_k \alpha_k(t+1) C(t+1) \delta f_k(t) - \rho_k \alpha_k(t+1) C(t+1) \delta g_k(t) u_d(t) - \\ - \rho_k \alpha_k(t+1) C(t+1) g_k(t) \delta u_k(t) + \rho_k \alpha_k(t+1) \zeta_k(t+1).\end{gathered} \tag{10}$$

Considering that $\zeta_k(t+1)$ is independent of $\alpha_k(t+1)$, and the norm is taken from both sides of (10), we have

$$\begin{gathered}|\delta u_{k+1}(t)| \le |1 - \rho_k C(t+1) g_k(t)| \, |\delta u_k(t)| + |\rho_k| \, |C(t+1)| \, |g_k(t)| \, |\delta u_k(t)| + \\ + |\rho_k| \, |\alpha_k(t+1)| \, |C(t+1)| \, |\delta f_k(t)| + |\rho_k| \, |\alpha_k(t+1)| \, |C(t+1)| \, |\delta g_k(t)| \, |u_d(t)| + \\ + |\rho_k| \, |\alpha_k(t+1)| \, |C(t+1)| \, |g_k(t)| \, |\delta u_k(t)| + |\rho_k| \, |\alpha_k(t+1)| \, |\zeta_k(t+1)| \quad \text{a.s.}\end{gathered} \tag{11}$$

We can prove $\lim\limits_{k\to\infty} \delta u_k(t) = 0$, $\forall t$ by mathematical induction.

Initial step. Assume that $t = 0$.

$$\begin{gathered}|\delta u_{k+1}(0)| \le |1 - \rho_k C(1) g_k(0)| \, |\delta u_k(0)| + |\rho_k| \, |C(1)| \, |g_k(0)| \, |\delta u_k(0)| + \\ + |\rho_k| \, |\alpha_k(1)| \, |C(1)| \, |\delta f_k(0)| + |\rho_k| \, |\alpha_k(1)| \, |C(1)| \, |\delta g_k(0)| \, |u_d(0)| + \\ + |\rho_k| \, |\alpha_k(1)| \, |C(1)| \, |g_k(0)| \, |\delta u_k(0)| + |\rho_k| \, |\alpha_k(1)| \, |\zeta_k(1)| \;\; \text{a.s.}\end{gathered} \tag{12}$$

According to A4, for sufficiently large k, we conclude that $C(1) g_k(0) > \psi$, where ψ is a suitable constant. Note that $\alpha_k(1)$ and $|C(1)|$ are bounded. Considering A5, we have $\delta x_k(0) \to 0$, therefore, according to A3, we conclude that $\delta f_k(0) \to 0$. Hence, $|\rho_k| \, |\alpha_k(1)| \, |C(1)| \, |\delta f_k(0)| \to 0$ w.p.1, when $k \to \infty$. In (12), the initial desired input vector is $u_d(0)$, therefore, the norm of $u_d(0)$ is bounded. Hence, concerning $\lim\limits_{k\to\infty} \rho_k = 0$, we conclude that $|\rho_k|\,|\alpha_k(1)|\,|C(1)|\,|\delta g_k(0)|\,|u_d(0)| \to 0$ w.p.1, when $k \to \infty$. Also, $\delta u_k(0)$ is the input error vector, therefore, its norm is bounded. Therefore, we have $|\rho_k|\,|C(1)|\,|g_k(0)|\,|\delta u_k(0)| \to 0$ and $|\rho_k|\,|\alpha_k(1)|\,|C(1)|\,|g_k(0)|\,|\delta u_k(0)| \to 0$, w.p.1, when $k \to \infty$. $\zeta_k(1)$ is a continuous function of white noise on $[0,\, N]$, therefore, $|\zeta_k(1)|$ is bounded. Therefore, concerning $\lim\limits_{k\to\infty} \rho_k = 0$, we have $|\rho_k|\,|\alpha_k(1)|\,|\zeta_k(1)| \to 0$, w.p.1, when $k \to \infty$.

We set $\sigma_1 = |1 - \rho_k C(1) g_k(0)|$, $\sigma_i = 0$, $i = 2,\, 3,\, ...,\, e_k = |\delta u_k(0)|$, and $\varphi_k = 0$, concerning the assumption of Theorem 1 and Lemma 1 of the paper [14], from inequality (12), we result $\lim\limits_{k\to\infty} |\delta u_k(0)| = 0$, w.p.1.

Inductive step. It is assumed that $\delta u_k(m) \to 0$ is true for $m = 0, 1, ..., t-1$, then we show $\delta u_k(m) \to 0$ for $m = t$. $\alpha_k(t+1)$ and $|C(t+1)|$ are bounded. In (11), concerning the inductive assumption and Lemma 1, we conclude that $\delta x_k(t) \to 0$ and $\delta f_k(t) \to 0$. Hence, in (11) we have $|\rho_k|\,|\alpha_k(t+1)|\,|C(t+1)|\,|\delta f_k(t)| \to 0$ w.p.1 when $k \to \infty$.

Note that $u_d(t)$ is the desired input vector, therefore, its norm is bounded. Hence, concerning $\lim\limits_{k\to\infty} \rho_k = 0$, we conclude that $|\rho_k|\,|\alpha_k(t+1)|\,|C(t+1)|\,|\delta g_k(t)|\,|u_d(t)| \to 0$ w.p.1, when $k \to \infty$. Also, $\delta u_k(t)$ is the input error vector, therefore, its norm is bounded. Therefore, we conclude that $|\rho_k|\,|C(t+1)|\,|g_k(t)|\,|\delta u_k(t)| \to 0$ and $|\rho_k|\,|\alpha_k(t+1)|\,|C(t+1)|\,|g_k(t)|\,|\delta u_k(t)| \to 0$, w.p.1, when $k \to \infty$.

$|\zeta_k(t+1)|$ is bounded, since $\zeta_k(t+1)$ is a continuous white noise function on $[0,\, N]$. Concerning $\lim\limits_{k\to\infty} \rho_k = 0$, we have $|\rho_k|\,|\alpha_k(t+1)|\,|\zeta_k(t+1)| \to 0$ w.p.1, when $k \to \infty$.

Let $\sigma_1 = |1 - \rho_k C(t+1) g_k(t)|$, $\sigma_i = 0$, $i = 2,\, 3,\, ...,\, e_k = |\delta u_k(t)|$, and $\varphi_k = 0$, concerning $|1 - \rho_k C(t+1) g_k(t)| < 1$, Lemma 1 of the paper [14], and (13), we conclude that $\lim\limits_{k\to\infty} |\delta u_k(t)| = 0$, w.p.1. □

Therefore, in the ILC update law (4), (5), we proved that the input error converges to zero w.p.1 when $k \to \infty$.

Although the system dynamics are assumed to be non-differentiable, there is no disturbance in proving the convergence of the algorithm (4), (5), and the GLC condition is sufficient to prove the convergence of the algorithm.

3. Numerical Example

In this section, in order to demonstrate the convergence feature of the proposed algorithm (4), (5), we consider the following stochastic nonlinear networked system that have GLC condition:

$$\begin{gathered} x_k^1(t+1) = x_k^1(t)\sin(|x_k^2(t) - 3|) + \tfrac{1}{2}\sin(|x_k^1(t) - 3|)u_k(t), \\ x_k^2(t+1) = 0,3\cos(t)\cos(x_k^1(t)) + 0,75\,u_k(t), \\ y_k(t) = 0,2x_k^1(t) + 0,35\,t^{0,15}x_k^2(t) + \zeta_k(t). \end{gathered} \tag{13}$$

Here $u_k(t)$ is the input, $y_k(t)$ is the output, and $\begin{bmatrix} x_k^1(t) \\ x_k^2(t) \end{bmatrix}$ is the state vector of system (13). The measurement noise of system (13) is $\zeta_k(t)$ with normal distribution $N(0,\, 0,01^2)$. The time interval is $[0, 60]$, and the desired output is $y_d(t) = 0,85\sin(\frac{\pi}{20}t) + 0,3\sin(\frac{\pi}{15}t)$. The initial iteration input signal is $u_1(t) = 0$. The initial state is $x_k^1(0) = x_k^2(0) = 0$. In the following, the convergence characteristics of the model (4), (5) and tracking performances

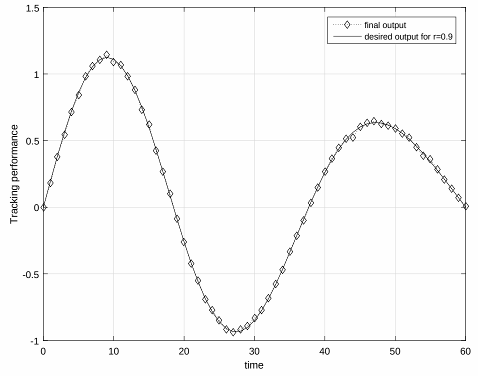

Fig. 2. Tracking performances of system (13) for $r = 0,9$

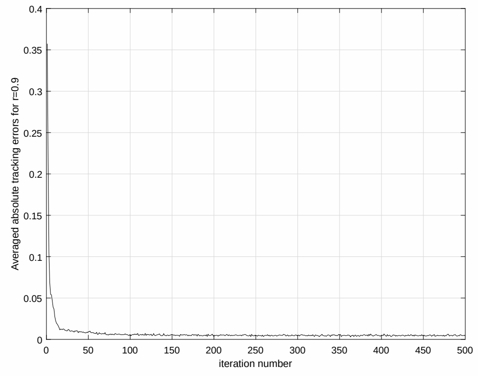

Fig. 3. Average absolute tracking errors of system (13) for $r = 0,9$

are examined according to the different probabilities of lost data. To evaluate tracking performance, the proposed algorithm is executed for 500 repetitions.

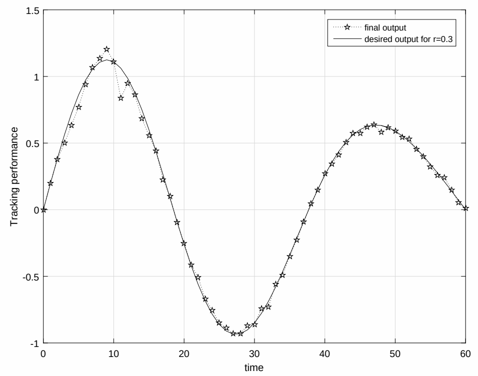

Fig. 4. Tracking performances of system (13) for $r = 0,3$

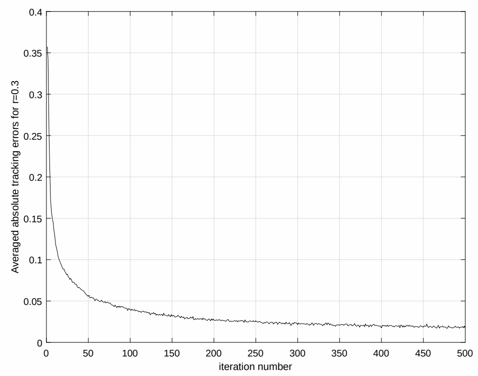

Fig. 5. Average absolute tracking errors of system (13) for $r = 0,3$

First, we set $r = 0,9$. Figure 2 shows tracking performance of system (13) for $r = 0,9$. As can be observed, the final output almost corresponds to the desired output.

Figure 3 presents the average absolute tracking error of outputs for $r = 0,9$. Figure 2 and Figure 3 indicate that the proposed algorithm has good tracking performances and is effective.

In the other case, consider $r = 0,3$. Figure 4 presents the tracking performance of the system for $r = 0,3$. Figure 5 shows the average absolute tracking error of outputs for $r = 0,3$. As it is observed, the probability of successful transfer of data for $r = 0,30$ is low, and update law performance is worse than for the case $r = 0,9$. But while the probability of successful transmission of data is low, performance is not bad.

Then, we set $r = 0,9, 0,7, 0,5$, and $0,3$, for the examination of the influence of various probabilities of lost data. As can be observed in Figure 6 and Figure 7, the algorithm almost retains its good performance, even if the probability of successful data transfer decreases with the increasing number of iterations.

In this example, we found that the non-differentiability of system did not disrupt the convergence and performance of the proposed algorithm, and the GLC condition is sufficient.

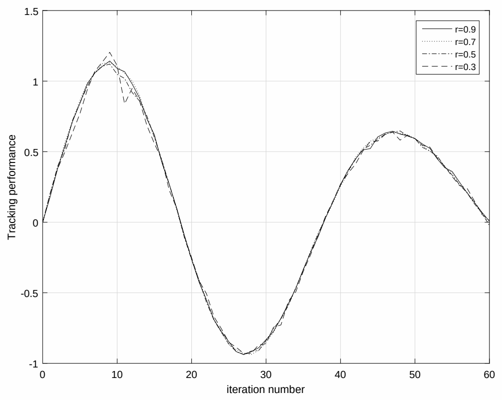

Fig. 6. Final outputs of system (13) for $r = 0,9, r = 0,7, r = 0,5, r = 0,3$

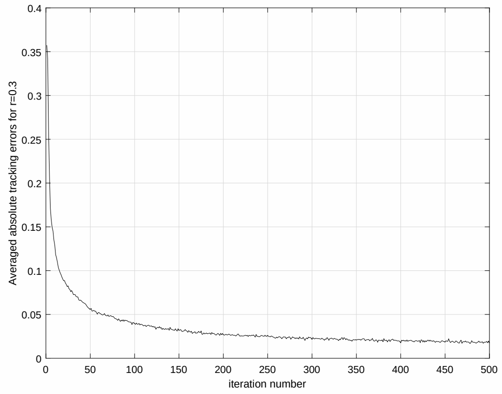

Fig. 7. Average absolute tracking error of system (13) for $r = 0,9, r = 0,7, r = 0,5, r = 0,3$

Conclusion

Differentiability of a system is one of the main assumptions for designing ILC algorithms in stochastic nonlinear networked systems. In this paper, in cases where stochastic nonlinear networked systems have non-differentiable dynamics, we designed the ILC algorithm for such systems based on the GLC condition.

We examined the analysis of convergence and the tracking performance investigation of the introduced algorithm. In this paper, in these stochastic nonlinear networked systems that have GLC condition, the random lost data was set in the measurement side. We modelled the random lost data by random Bernoulli variables.

As it turned out, there is no restricted condition for the stochastic lost data probabilities in the convergence investigation of the input error. Also, we showed that to present the ILC algorithm, differentiability of the system dynamics is not necessary, and the GLC condition is enough for generating the ILC algorithm for stochastic nonlinear networked systems. We showed that, in the ILC update law (4), (5), if $|1 - \rho_k C(t+1) g_k(t)| < 1$, then the input error converges to zero in the almost sure sense.

The theoretical conclusions were confirmed by a numerical example.

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УПРАВЛЕНИЕ НЕЛИНЕЙНЫМИ СТОХАСТИЧЕСКИМИ СЕТЕВЫМИ СИСТЕМАМИ С НЕДИФФЕРЕНЦИРУЕМОЙ ДИНАМИКОЙ С ИТЕРАТИВНЫМ ОБУЧЕНИЕМ

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При разработке алгоритма управления с итеративным обучением (ILC) для стохастических нелинейных сетевых систем основным предположением является дифференцируемость динамики системы. Во многих случаях в действительности стохастические нелинейные сетевые системы обладают недифференцируемой динамикой, но их динамические функции после дискретизации с использованием обычных методов имеют глобальное непрерывное условие Липшица (GLC). В этой статье мы применяем алгоритм ILC для стохастических нелинейных сетевых систем, которые имеют условие GLC. Мы демонстрируем, что для разработки алгоритма ILC дифференцируемость динамики системы не требуется, а условие GLC достаточно для разработки алгоритма ILC для стохастических нелинейных сетевых систем с недифференцируемой динамикой. Мы исследуем анализ сходимости и отслеживаемость предложенного обновленного закона для стохастических нелинейных сетевых систем с условием GLC. Мы показываем, что не существует ограниченного условия для вероятностей выпадения стохастических данных при исследовании сходимости входной ошибки. Затем результаты рецензируются и подтверждаются численным примером.

Ключевые слова: управление с итеративным обучением; стохастическая нелинейная сетевая система; недифференцируемый; глобальное непрерывное Липшица (GLC); пропадание данных.

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