КРАТКИЕ СООБЩЕНИЯ

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STABILITY OF A STATIONARY SOLUTION TO ONE CLASS OF NON-AUTONOMOUS SOBOLEV TYPE EQUATIONS

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The article is devoted to the study of the stability of a stationary solution to the Cauchy problem for a non-autonomous linear Sobolev type equation in a relatively bounded case. Namely, we consider the case when the relative spectrum of the equation operator can intersect with the imaginary axis. In this case, there exist no exponential dichotomies and the second Lyapunov method is used to study stability. The stability of stationary solutions makes it possible to evaluate the qualitative behavior of systems described using such equations. In addition to introduction, conclusion and list of references, the article contains two sections. Section 1 describes the construction of solutions to non-autonomous equations of the class under consideration, and Section 2 examines the stability of a stationary solution to such equations.

Keywords: relatively bounded operator; Lyapunov's second method; local stream of operators; asymptotic stability.

Introduction

The study of the stability of stationary solutions to abstract operator-differential equations allows us to obtain results on the qualitative behavior of a system described using equations of this type [1, 2]. Using such results for specific models, it is possible to obtain values of characteristics that guarantee the stability of the simulated system.

In this article we investigate the properties of stationary solutions for a system of non-autonomous linear equations of the Sobolev type

$$L\dot{u}(t) = \alpha(t)Mu(t) + f(t), \quad \ker L \neq \{0\}, \tag{1}$$

where L and M are linear bounded operators acting from the space $\mathfrak{U}$ to the space $\mathfrak{F}$, a vector-function $f : \mathbb{R} \to \mathfrak{F}$ characterizes the external impact on the system, and a scalar function $\alpha : [0, T] \to \mathbb{R}_+$ characterizes the change in time of the parameters of this system. Here and below $\mathfrak{U}$ and $\mathfrak{F}$ are some Banach spaces. The properties of solutions to non-autonomous equations resolved with respect to the time derivative were studied, for example, in [3]. Sobolev type equations are understood as equations unresolved with respect to the highest derivative [1, 2, 4, 5]. A characteristic feature of such equations is the fundamental unsolvability of the Cauchy problem

$$u(0) = u_0$$

with an arbitrary initial data u_0, which can be even from a dense set in $\mathfrak{U}$ [1, 6]. For the existence of solutions to the Sobolev type equation, it is necessary that the initial data belong to some set of permissible initial values, which is understood as the phase space of these equations [7].

The solvability of non-autonomous Sobolev type equations of the form (1) was first considered in [8] and the proposed methods were applied to study various problems. To construct a solution to non-autonomous equation (1), we use the technique proposed in [9]. The stability of solutions to Sobolev type equations with constant coefficients was studied by many authors, more details on this can be found in [8] and [10]. In this paper, the stability of a stationary solution to equation (1) is considered under general assumptions about the position of its relative spectrum. Thus, exponential dichotomies of solutions [8] do not necessarily exist for it, and therefore we use the second Lyapunov method [10, 11] to study the stability.

1. Solvability of Non-Autonomous Sobolev Type Equation

Let $\mathfrak{U}$ and $\mathfrak{F}$ be Banach spaces, the operators $L \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ (linear and continuous) and $M \in \mathcal{C}l(\mathfrak{U};\mathfrak{F})$ (linear, closed and densely defined).

Consider an *L-resolvent set* $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{U})\}$ and an *L-spectrum* $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ of the operator M ([5], par. 2.1). The set $\rho^L(M)$ is always open, so the L-spectrum $\sigma^L(M)$ is always closed. Also, the operator $(\mu L - M)^{-1}$ is a holomorphic function of the variable μ on the set $\rho^L(M)$.

Definition 1. [7] The operator M is called *spectral bounded relatively to the operator L* (or simlpy *(L,σ)-bonded*), if $\exists a > 0 \quad \forall \mu \in \mathbb{C} \quad (|\mu| > a) \Rightarrow (\mu \in \rho^L(M))$.

Let the operator M be (L,σ)-bounded then we choose the loop $\gamma=\{\mu\in\mathbb{C} : |\mu|=r>a\}$ and construct the operators $P = \frac{1}{2\pi i}\int_\gamma R^L_\mu(M)d\mu$ and $Q = \frac{1}{2\pi i}\int_\gamma L^L_\mu(M)d\mu$, where the operator $R^L_\mu(M) = (\mu L - M)^{-1}L$ is a *right L-resolvent of the operator M*, and the operator $L^L_\mu(M) = L(\mu L - M)^{-1}$ is a *left L-resolvent of the operator M*. Here the integrals are understood as Riemann integrals. So the operators $P \in \mathcal{L}(\mathfrak{U})$ and $Q \in \mathcal{L}(\mathfrak{F})$.

Lemma 1. *Let the operator M be (L,σ)-bounded then the operators $P \in \mathcal{L}(\mathfrak{U})$ and $Q \in \mathcal{L}(\mathfrak{F})$ are projectors.*

Denote $\mathfrak{U}^0 = \ker P$, $\mathfrak{F}^0 = \ker Q$, $\mathfrak{U}^1 = \mathrm{im} P$, $\mathfrak{F}^1 = \mathrm{im} Q$. Denote the restriction of the operator L to the set $\mathfrak{U}^k$ by L_k, and the restriction of the operator M to the set $\mathrm{dom} M \cap \mathfrak{U}^k$, $k = 0, 1$, by M_k. Due to the properties of operators, linear sets $\mathrm{dom} M_k = \mathrm{dom} M \cap \mathfrak{U}^k$ are dense in $\mathfrak{U}^k$, $k = 0, 1$.

Theorem 1. [7] **(Sviridyuk's splitting theorem)**
Let the operator M be (L,σ)-bounded then
(i) *the operators $L_0 \in \mathcal{L}(\mathfrak{U}^0;\mathfrak{F}^0)$ and $L_1 \in \mathcal{L}(\mathfrak{U}^1;\mathfrak{F}^1)$;*
(ii) *the operators $M_0 \in \mathcal{C}l(\mathfrak{U}^0;\mathfrak{F}^0)$ and $M_1 \in \mathcal{L}(\mathfrak{U}^1;\mathfrak{F}^1)$;*
(iii) *there exist the operator $L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1;\mathfrak{U}^1)$ and $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0;\mathfrak{U}^0)$.*

Denote $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{U}^0)$, $S = L_1^{-1}M_1 \in \mathcal{L}(\mathfrak{U}^1)$.

Definition 2. For the L-resolvent $(\mu L - M)^{-1}$ of the operator M, the infinity point is called

(i) a *disposable singular point*, if $H = \mathbb{O}$;
(ii) a *polar* of the order $p \in \mathbb{N}$, if $H^p \neq \mathbb{O}$ and $H^{p+1} = \mathbb{O}$;
(iii) an *essentially singular point*, if $H^p \neq \mathbb{O}$ for all $p \in \mathbb{N}$.

Remark 1. For further discussion, it is more convenient to refer to the disposable singular point as a "pole of the zero order". Then the operator M is called (L, p)-bounded, $p \in \{0\} \cup \mathbb{N} \equiv \mathbb{N}_0$, if M is (L, σ)-bounded, and the point ∞ is a pole of the order $p \in \mathbb{N}_0$ of its L-resolvents.

A vector-function $u \in C^\infty(\mathbb{R}; \mathfrak{U})$ is called a *solution* $u = u(t)$ to the equation

$$L\dot{u} = Mu, \tag{2}$$

if u satisfies this equation. The solution $u(t)$ to equation (2) is called the *solution to the Cauchy problem for equation* (2) if u additionally satisfies the *Cauchy condition*

$$u(0) = u_0 \tag{3}$$

for some vector $u_0 \in \mathfrak{U}$.

Definition 3. A set $\mathfrak{P}$ is called *a phase space* of equation (2), if
(i) any solution $u = u(t)$ of (2) belongs to $\mathfrak{P}$ as a trajectory (i.e. $u(t) \in \mathfrak{P} \ \ \forall t \in \mathbb{R}$);
(ii) for any $u_0 \in \mathfrak{P}$ there exists a unique solution to problem (2), (3).

Theorem 2. [7] *Let the operator M be (L, p)-bounded ($p \in \mathbb{N}_0$) then the phase space of* (2) *is the subspace $\mathfrak{U}^1$.*

Definition 4. [5] We refer to the operator-function $U^\cdot \in C^\infty(\mathbb{R}; \mathfrak{U})$ as a *group of resolving operators* (or, in short, as a *group*) of equation (2) if
(i) $U^s U^t = U^{s+t}$ for all $s,\, t \in \mathbb{R}$;
(ii) for any $u_0 \in \mathfrak{U}$ the vector-function $u(t) = U^t u_0$ is a solution to (2).

Let us identify the group and its graph $\{U^t : t \in \mathbb{R}\}$. The group $\{U^t : t \in \mathbb{R}\}$ is called *holomorphic* if $\{U^t : t \in \mathbb{R}\}$ is analytically continuous into the entire complex plane $\mathbb{C}$ under Properties (i), (ii); and is *degenerate* if its unit U^0 is a projector. For a holomorphic degenerate group, the concepts of *kernel* and *image* are correct, and $\ker U^\cdot = \ker U^0 = \ker U^t$ for any $t \in \mathbb{R}$, and $\operatorname{im} U^\cdot = \operatorname{im} U^0 = \operatorname{im} U^t$ for any $t \in \mathbb{R}$. A holomorphic degenerate group $\{U^t : t \in \mathbb{R}\}$ is called the resolving group of equation (2) if its image $\operatorname{im} U^\cdot$ coincides with the phase space of equation (2).

Theorem 3. [7] *Let the operator M be (L, p)-bounded ($p \in \mathbb{N}_0$) then there exists a unique resolving group of equation* (2)*, which has the form*

$$U^t = \frac{1}{2\pi i} \int\limits_\gamma R^L_\mu(M) e^{\mu t} d\mu, \ t \in \mathbb{R}, \tag{4}$$

where $\gamma = \{\mu \in \mathbb{C} : |\mu| = r > a\}$ is the closed loop.

Remark 2. It is clear that the identity of group (4) is $U^0 = P$.

On the interval $\mathfrak{J} \subset \mathbb{R}$, consider the Cauchy problem ($t_0 \in \mathfrak{J}$)

$$u(t_0) = u_0, \tag{5}$$

for the homogeneous non-autonomous equation

$$L\dot{u}(t) = \alpha(t)Mu(t). \tag{6}$$

Definition 5. [9] The vector-function $u \in C^1(\mathfrak{J};\mathfrak{U})$ is called a *solution* to equation (6) if u satisfies this equation on $\mathfrak{J}$. A solution to (6) is called *a solution to the Cauchy problem* (5), (6), if it additionally satisfies condition (5).

Theorem 4. [9] *Let the operator M be (L,p)-bounded ($p \in \mathbb{N}_0$) and the function $\alpha \in C(\mathbb{R},\mathbb{R}_+)$ then a phase space of equation* (6) *is the subspace $\mathfrak{U}^1$.*

Definition 6. [9] The two-parameter family $U(\cdot,\cdot) : \mathbb{R}\times\mathbb{R} \to \mathcal{L}(\mathfrak{U})$ is called a *family of resolving operators*, if the following conditions hold :

(i) $U(t,t) = P$ for all $t \in \mathbb{R}$;

(ii) $U(t,s)U(s,\tau) = U(t,\tau)$ for all $t,\tau,s \in \mathbb{R}$.

A family of resolving operators is called *analytic* if its operators admit an analytic continuation into the entire complex plane $\mathbb{C}$ under Properties (i) and (ii) from Definition 6. The two-parameter family of operators $U(\cdot,\cdot) : \mathbb{R}\times\mathbb{R} \to \mathcal{L}(\mathfrak{U})$ is called a *family of resolving operators* of equation (6) if for any $u_0 \in \mathfrak{U}$ the vector function $u(t) = U(t,t_0)u_0$ is a solution to equation (6) (in the sense of Definition 5).

Let the operator M be (L,p)-bounded ($p \in \mathbb{N}_0$) and the function $\alpha \in C(\mathbb{R};\mathbb{R})$. By analogy with group (4), consider the operators

$$U(t,s) = \frac{1}{2\pi i}\int\limits_{\gamma} R^L_\mu(M)\exp\left(\mu\int\limits_s^t \alpha(\zeta)d\zeta\right)d\mu, \qquad s<t \tag{7}$$

with $s,t \in \mathbb{R}$ and the closed loop $\gamma = \{\mu \in \mathbb{C} : |\mu| = r > a\}$

Theorem 5. [9] *Let the operator M be (L,p)-bounded ($p \in \mathbb{N}_0$) and the function $\alpha \in C(\mathbb{R};\mathbb{R})$, then the family of operators $\{U(t,s) \in \mathcal{L}(\mathfrak{U}) : \ t,s \in \mathbb{R}\}$, given by formula* (7), *is an analytic degenerate family of resolving operators.*

Finally, we describe the solution for the inhomogeneous equation

$$L\dot{u}(t) = \alpha(t)Mu(t) + g(t), \tag{8}$$

where $\alpha : [0,T] \to \mathbb{R}_+$ is a scalar function that characterizes the change in time of the parameters of the mutual influence of the states of the system under study, the vector function $g : [0,T] \to \mathfrak{F}$ characterizes the external impact. Denote $(\mathbb{I}_{\mathfrak{Y}} - Q)g(t) = g^0(t)$.

Theorem 6. [9] *Let $[0,T] \in \mathfrak{J}$, the operator M be (L,p)-bounded ($p \in \mathbb{N}_0$) and the function $\alpha \in C^{p+1}([0,T];\mathbb{R}_+)$. Then for an arbitrary vector-function $g : [0,T] \to \mathfrak{F}$ such that $Qg \in C^1([0,T];\mathfrak{F}^1)$, $g^0 \in C^{p+1}([0,T];\mathfrak{F}^0)$ and under the condition of approval*

$$(\mathbb{I}_{\mathfrak{U}} - P)u_0 = -\sum_{k=0}^{p} H^k M_0^{-1}\left(\frac{1}{\alpha(0)}\frac{d}{dt}\right)^k \frac{g^0(0)}{\alpha(0)},$$

and for any initial data $u_0 \in \mathfrak{U}$ *there exists a unique solution* $u \in C^1([0,T];\mathfrak{U})$ *to Cauchy problem* (5), (8), *which has the form*

$$u(t) = U(t,0)Pu_0 + \int_0^t U(t,s)L_1^{-1}Qg(s)ds - \sum_{k=0}^{p} H^k M_0^{-1} \left(\frac{1}{\alpha(t)}\frac{d}{dt}\right)^k \frac{g^0(t)}{\alpha(t)}. \qquad (9)$$

2. Second Lyapunov Method in Normed Spaces

Let $\mathfrak{V}$ be a normed space.

Definition 7. A *local two-parameter stream* on $\mathfrak{V}$ (in shortly, *stream*) is a map S such that for all $u \in \mathfrak{V}$ and some $\tau = \tau(u) \in \mathbb{R}_+$, the following conditions hold:

(i) $S = S_s^t u \in \mathfrak{V}$ for all $t, s \in (-\tau;\tau)$; $S_0^0 u = u$;

(ii) $S_s^t = S_z^t S_s^z u$ for all $t, s, z \in (-\tau, \tau)$.

A point $u \in \mathfrak{V}$ such as

(iii) $S_s^t u = u$ for all $t, s \in (-\tau;\tau)$,

is called a *stationary point* of the stream S.

Definition 8. A stationary point $u \in \mathfrak{V}$ of the stream S is called

(i) *stable* (according to Lyapunov), if for any neighborhood $\mathfrak{O}_u$ of the point $u \in \mathfrak{V}$ there exists a neighborhood $\mathfrak{O}'_u$ (may be, another) of this point such that $S_s^t v \in \mathfrak{O}'_u$ for all $v \in \mathfrak{O}_u$ and $t, s \in \mathbb{R}_+$;

(ii) *asymptotically stable* (according to Lyapunov), if it is stable and for any point v from some neighborhood $\mathfrak{O}_u$ of the point u the following is true: $S_s^t v \to u$ with $t \to \infty$.

Definition 9. A functional $V \in C(\mathfrak{V};\mathbb{R})$ is called *Lyapunov functional* of the stream S, if for all $u \in \mathfrak{V}$ it has the form $\dot{V}(u) = \overline{\lim_{t\to 0+}} \dfrac{(V(S_0^t u) - V(u))}{t} \leq 0$.

Theorem 7. *Let* $u \in \mathfrak{V}$ *be the stationary point of the stream* S *on* $\mathfrak{V}$. *This point* $u \in \mathfrak{V}$ *is stable if for the stream* S *there exists a Lyapunov functional, which satisfies the following two conditions:*

(i) $V(u) = 0$;

(ii) $V(v) \geq \varphi(\|v-u\|)$ *with some strictly increasing continuous function* φ *such that* $\varphi(0) = 0$ *and* $\varphi(r) > 0$ *for* $r \in \mathbb{R}_+$.

Proof. We follow [10, 11], where a similar theorem is proved in the case of a stationary Sobolev type equation.

So, for every $r \in \mathbb{R}_+$ we put $\mathfrak{O}_r = \{v \in \mathfrak{V} : V(v) < r\}$. Each of the sets $\mathfrak{O}_r$ is a neighborhood of the point u, and $V \in \mathfrak{O}_r \Rightarrow V(S_0^t v) \leq V(v) < r$ for all $t \in \mathbb{R}_+$.

If $V(v) \geq \varphi(\|v-u\|)$ then for any $\varepsilon \in \mathbb{R}_+$ there exists $r = \varphi(\varepsilon) > 0$ such that $V(v) < r \Rightarrow \|v-u\| < \varepsilon$. Due to the continuity of V, there exists $\delta \in \mathbb{R}_+$ such that with $\|v-u\| < \delta$ we have $v \in \mathfrak{O}_r$, and we get $S_0^t v \in \mathfrak{O}_r$ such that $\|S_0^t v\| \leq \varepsilon$ for all $t \in \mathbb{R}_+$.

□

Theorem 8. *Let the conditions of Theorem* 7 *be fulfilled, and suppose that there exists a strictly increasing continuous function* ψ *such that* $\psi(0) = 0$ *and* $\psi(r) > 0$ *for* $r \in \mathbb{R}_+$, *and* $\dot{V}(v) \leq -\psi(\|v-u\|)$, *then the point* $u \in \mathfrak{V}$ *is asymptotically stable.*

Proof. Let the point be $v \in \mathfrak{O}_u$, then by virtue of Theorem 7 $V(S_0^t v)$ is a non-increasing non-negative function with $t \in \mathbb{R}_+$.

Let $l = \lim\limits_{t\to+\infty} V(S_0^t v)$ and suppose that $l > 0$, then $\inf\limits_{t\in\mathbb{R}_+} \|S_0^t v\| > 0$. And we can conclude that $\sup\limits_{t\in\mathbb{R}_+} \dot{V}(S_0^t v) \leq -m$ for some $m \in \mathbb{R}_+$, which contradicts the non-negativity of $V(S_0^t v)$. So $V(S_0^t v)$ and $\|S_0^t v\|$ tend to zero at $t \to +\infty$.

□

Theorem 9. *Let the operator M be (L,p)-bounded $(p \in \mathbb{N}_0)$ and the function $\alpha \in C(\mathbb{R};\mathbb{R})$, then a family of operators $\{S_s^t u \in \mathcal{L}(\mathfrak{U}) : \ t, s \in \mathbb{R}\}$ is a local stream of operators. The zero point is the stationary point of this stream. And the operators of this stream have the form*

$$S_s^t = \frac{1}{2\pi i}\int_\gamma R_\mu^L(M)\exp\left(\mu\int\limits_s^t \alpha(\zeta)d\zeta\right)d\mu, \quad s,t\in\mathbb{R}, \quad s=0<t, \tag{10}$$

where the closed contour γ bounds the area containing the L-spectrum $\sigma^L(M)$ of the operator M.

Proof. It is obvious that the zero point is a stationary point of this stream due to the linearity of its operators.

Let us show that $S_s^t u$ is a local stream of operators. Statement (i) of Definition 7 follows from the method of specifying the operator $S_s^t g$ using (7), Property (i) of Definition 6 and Remark 2.

We show the fulfillment of Statement (ii) from Definition 7. To do this, consider

$$S_z^t S_s^z u = \frac{1}{(2\pi i)^2}\int\limits_\gamma R_\mu^L(M)\exp\left(\mu\int\limits_z^t\alpha(\zeta)d\zeta\right)d\mu\int\limits_{\gamma'} R_\lambda^L(M)u\exp\left(\lambda\int\limits_s^z\alpha(\zeta)d\zeta\right)d\lambda =$$

$$= \frac{1}{(2\pi i)^2}\int\limits_\gamma\left(\int\limits_{\gamma'} R_\mu^L(M)R_\lambda^L(M)u\exp\left(\lambda\int\limits_z^t\alpha(\zeta)d\zeta\right)d\lambda\right)\exp\left(\mu\int\limits_s^z\alpha(\zeta)d\zeta\right)d\mu =$$

$$= \frac{1}{(2\pi i)^2}\left(\int\limits_\gamma\int\limits_{\gamma'}\frac{\exp\left(\lambda\int\limits_z^t\alpha(\zeta)d\zeta\right)d\lambda}{\lambda-\mu}R_\mu^L(M)u\exp\left(\mu\int\limits_s^z\alpha(\zeta)d\zeta\right)d\mu + \right.$$

$$\left. + \int\limits_{\gamma'} R_\lambda^L(M)u\exp\left(\lambda\int\limits_z^t\alpha(\zeta)d\zeta\right)\int\limits_\gamma\frac{\exp\left(\mu\int\limits_s^z\alpha(\zeta)d\zeta\right)d\mu}{\mu-\lambda}d\lambda\right),$$

where the point $\mu \in \gamma$ lies inside the area bounded by the contour γ', and the point $\lambda \in \gamma'$ is located outside the area bounded by the contour γ. Then by Deduction Theorem

$$\int\limits_{\gamma'}\frac{\exp\left(\lambda\int\limits_z^t\alpha(\zeta)d\zeta\right)d\lambda}{\lambda-\mu} = 2\pi i\exp\left(\mu\int\limits_z^t\alpha(\zeta)d\zeta\right), \qquad \int\limits_\gamma\frac{\exp\left(\mu\int\limits_s^z\alpha(\zeta)d\zeta\right)d\mu}{\mu-\lambda} = 0,$$

and we have the fulfillment of Statement (ii) of Definition 7

$$S_z^t S_s^z u = \frac{1}{2\pi i} \int_\gamma \exp\left(\mu \int_z^t \alpha(\zeta) d\zeta\right) R_\mu^L(M) u \exp\left(\mu \int_s^z \alpha(\zeta) d\zeta\right) d\mu =$$

$$= \frac{1}{2\pi i} \int_\gamma R_\mu^L(M) u \exp\left(\mu \left(\int_z^t \alpha(\zeta) d\zeta + \int_s^z \alpha(\zeta) d\zeta\right)\right) d\mu = S_s^t u.$$

□

Thus, using a family of resolving operators, it is always possible to construct a local stream of operators in the sense of Definition 7. For a specific type of operators, using Theorems 7 and 8, based on information about the points of the relative spectrum $\sigma^L(M)$, it is possible to investigate stationary zero solutions for Lyapunov stability.

Conclusion

The results obtained in this paper are planned to be used to study the stability of the null solution in non-autonomous Hoff models on geometric graphs. These models describe structures made of I-beams. In such models, at high temperatures, the parameter on the right side of the equation ceases to be stationary, which explains the appearance of the time function in the equation. The stability of the solution of such models makes it possible to more accurately determine the time of maintaining the stability of the structure, which is an urgent problem when carrying out work to eliminate fires.

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УСТОЙЧИВОСТЬ СТАЦИОНАРНОГО РЕШЕНИЯ ОДНОГО КЛАССА НЕАВТОНОМНЫХ УРАВНЕНИЙ СОБОЛЕВСКОГО ТИПА

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Статья посвящена исследованию устойчивости стационарного решения задачи Коши для неавтономного линейного уравнения соболевского типа в относительно ограниченном случае. А именно рассматривается случай, когда относительный спектр оператора уравнения может пересекаться с мнимой осью. В этом случае не существуют экспоненциальные дихотомии и для исследования устойчивости применяется второй метод Ляпунова. Устойчивость стационарных решений позволяет оценить качественное поведение систем, описываемых с помощью таких уравнений. Статья кроме введения, заключения и списка литературы содержит две части. В первой из них описывается построение решений неавтономных уравнений рассматриваемого класса, а во второй исследуется устойчивость стационарного решения таких уравнений.

Ключевые слова: относительно ограниченный оператор; второй метод Ляпунова; локальный поток операторов; асимптотическая устойчивость.

Литература

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