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FORECASTING STOCK RETURN VOLATILITY USING THE REALIZED GARCH MODEL AND AN ARTIFICIAL NEURAL NETWORK

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Volatility forecasting is required for risk management, asset allocation, option pricing, and financial market trading. It can be done by using various time series forecasting techniques and Artificial Neural Networks (ANN).

The current research focuses on the modeling and forecasting of stock market indices using high-frequency data. A recent high-frequency volatility model is called the Realized GARCH (RGARCH) model, where the key feature is an equation that relates the realized measure to the conditional variance of returns. This equation incorporates an asymmetric reaction to shocks, providing a highly flexible representation of market dynamics.

This paper proposes an hybrid model where ANN and RGARCH are used to forecast stock return volatility. This model was established by entering the predicted Realized Volatility (RV), calculated using RGARCH, into the ANN. The choice of the input variables of the ANN is made using the Granger causality test in order to reduce the noise which would affect the prediction system and which could be generated by an input variable not statistically linked to stock market volatility.

The results show that a hybrid model based on a recurrent neural network (RNN) outperforms the RGARCH and HAR-type models in out-of-sample evaluations according to the RMSE and the correlation coefficient.

Keywords: volatility; Realized GARCH model; hybrid; Granger causality test.

Introduction

Forecasting the stock market is an effort to anticipate how a future event will be unfolded and is done by using various time series forecasting techniques. The stock market is fundamentally volatile, therefore forecasting its movement will be beneficial to stock traders when developing trading methods. Researchers have used different forecasting techniques to examine the volatility in the stock market.

Engle [1], a pioneer in volatility modeling, developed the Auto Regressive Conditional Heteroskedastic (ARCH) model to anticipate time series data volatility. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, developed by Bollerslev [2], incorporates a moving average component in modeling time-series data volatility in addition to the autoregressive component. In these models, The asymmetrical return volatility is not taken into consideration. A number of articles argued that GARCH models have limited practical usefulness.

Nelson [3] proposed the exponential GARCH (EGARCH) model to describe the volatility of time-series data based on the asymmetrical influence of positive and negative error factors on volatility. The model anticipates the volatility of a time-series variable using conditional variance as a multiplicative function instead of the addictive functions of lagged innovations. It takes into account the symmetrical and asymmetrical volatility of returns. Eryilmaz [4] examined the stock market volatility of the Istanbul Stock Exchange using the BIST-100 index from 1997 to 2015. The study used the ARCH, GARCH, EGARCH, and Threshold ARCH models. The EGARCH model was the most accurate predictor of volatility.

A number of studies have been published in response to the emergence of high-frequency data in the financial world. The realized variance established by Anderson et al [5] and Bandorff-Nielsen and Shephard [6], the realized kernel presented by Barndorff-Nielsen et al (2008), and many other related quantities have become popular research tools. These measures are more precise and efficient on the level of daily volatility than the squared returns of financial series often used.

Engel (2002) was the first to examine this type of approach, by proposing the GARCH-X model which includes the realized variance in the GARCH model. Nielsen et al (2007), extended this model using realized variance. This model, however, is considered incomplete or partial, because it does not take into consideration the dynamics of the measurements made.

A recent study of high-frequency volatility modeling using a GARCH model was proposed by Hansen et al [7]. These authors built a new model called Realized GARCH (RGARCH). An equation that connects the realized measure to the conditional variance of returns is an important component of the RGARCH model. It is distinguished from the classic GARCH model by its ability to precisely determine the dynamics of its relation with conditional variance, taking into account the asymmetry of effects due to shocks.

In financial econometrics, the importance of jumps is rapidly growing. Barndorff-Nielsen and Shephard [8], Lee and Mykland [9], Ait-Sahalia and Mancini [10], and Boudt et al [11] have researched jump detection and volatility in the presence of jumps. Barndorff-Nielson and Sheppard (2004) suggest using the bipower variation measure (BPV) to separately estimate the integrate variance (IV) and the jump components.

In finance, the study of long-memory properties of time series is even more common. Several authors have suggested that the stock returns for stock prices exhibit long-memory behavior (Mandelbrot, 1971, Greene and Fielitz, 1977). Corsi [12] proposed the Heterogeneous Autoregressive model for RV (HAR-RV) that takes into account RV throughout a range of interval sizes. This model has been successful in capturing the long-memory behavior of volatility.

Financial series present complex and non-linear behaviors that make modeling difficult, AI techniques have been successfully tested for prediction problems. Artificial Neural Networks (ANN) have been used successfully in many studies [13]. Fausett [14] showed that an ANN with single hidden layer is sufficient to approximate any continuous function to an arbitrary degree of precision.

To improve the predictive ability of financial time-series models, traditional time-series models are combined with neural networks for volatility forecasting. Hybrid systems attempt to go beyond these results and overcome the shortcomings of other models by extracting input variables from statistical methods and including them in the ANN. Lu et al [15] evaluate the performance of two types of hybrid ANN and GARCH-type models in forecasting volatility showing that the EGARCH-ANN model outperforms other models in forecasting the log-return series volatilities in the Chinese energy market, and there are significant leverage effects in the Chinese energy market.

Dimitrios [16] investigates whether nonlinear models such as Principal Components Combining, neural networks, and GARCH are more accurate than the Heterogeneous Autoregressive (HAR) model at forecasting RV. The results show that the persistence of RV is just too significant to be ignored in RV forecasting.

[17] compares two approaches: HAR-RV and Feedforward Neural Networks (FNN). It was found that HAR-RV-J performs better, but not significantly better, than the FNN-HAR model in terms of accuracy, while the FNN-HAR-J model performs significantly better than the FNN-HAR model. Huang [18] proposed a network autoregressive model with GARCH effects (NAR-GARCH), which makes satisfactory predictions for 20 stock indices.

In this paper, we are interested in the statistical analysis of the history of stock index return volatility based on the analysis and modeling of the internal dynamics of the series using the information extracted from these statistical characteristics. The data used in this study is from the Oxford-Man Institute "realised library" which contains daily non-parametric measures of how volatile financial indexes were in the past.

The problem of predicting stock return volatility is widely documented in the literature. However, a literature review shows that selection criteria for inputs are rarely used. In general, a large set of predictor variables of different categories is considered without verifying whether these input variables cause variation in return volatilities. This approach may automatically include entries that introduce noise into the predictive systems and therefore reduce their predictive performance.

The most important step in a neural network is the right choice of input variables in order to provide predictive systems with only the input variables that show a statistical causal link with the output variable. The input variables chosen in this study are the only ones that cause a significant variation in the (RV) using the Granger causality test [19]. The proposed model blends the Realized GARCH model and ANN. The results show that the ANN models are more resilient than FNN and statistical models in predicting volatility.

The rest of this paper is divided into three sections. The Methodology is presented in section 1. The data analysis and application are presented in section 2, and concluding remarks are presented in section 3.

1. Methodology

The RGARCH model, introduced by Hansen et al (2012), describes the RV stylized facts very well. This shows that, compared to traditional GARCH models that simply by employing daily returns, a RGARCH structure leads to substantial improvements in the empirical fit. The model requires a good choice of input variables for the ANN that can lead to the best predictions. The choice of input variables is based on the statistical analysis of the history of the series in question. The variables selected by the Granger test will constitute the final inputs of the predictive system. The principal input in the proposed models is the forecast RV of the RGARCH model, as a result of their application in forecasting the volatilities of economic and financial variables, we also included the previous bipower and jump volatility components as ANN inputs to capture the impact of jumps. Finally, to capture the long memory in the series we included weekly and monthly volatility.

The goal is to compare various combinations of input variables in order to detect which ones generates noise in the predictive model. The back-propagation algorithm is used to estimate the weights of the hybrid models.

1.1. Realized Measure

Assume that the logarithm of an asset price p_t follows the diffusion equation:

$$p_t = \int_0^1 \mu_s ds + \int_0^1 \sigma_s dW_s, \tag{1}$$

where μ_s is the drift, σ_s is the spot volatility and W_s is the standard Brownian motion, with the time interval normalized to 1.

The quadratic variation (QV)defined as :

$$QV = \int_0^1 \sigma_s^2 ds. \tag{2}$$

The sum of frequently sampled squared returns, also known as RV, is a natural estimator for QV (Andersen et al 2001).

Define $r_{i,t} = p_{i,t} - p_{i-1,t}$ as the i^{th} return on day t, where each intra-day time is subscripted as $i = 1, 2, ..., n$. The realized variance is simply the sum of the n intra-day squared returns

$$RV_t = \sum_{i=1}^{n} r_{i,t}^2. \tag{3}$$

This estimator converges to QV,

$$RV \xrightarrow{p} QV. \tag{4}$$

1.2. Jump Process and Bipower Variation

When unexpected news hits the market, prices tend to show sudden and distinct movements, i.e. jumps.

Let p_t denote the logarithmic asset price at time t. The price in stochastic differential equation form is:

$$p_t = \int_0^1 \mu_s ds + \int_0^1 \sigma_s dW_s + \sum_{0 \leq s \leq N} J_s, \tag{5}$$

where N is the number of jumps and is a finite-activity simple counting process and J_s are non-zero random variables.

The QV of returns over the interval [0,1] is given by the sum of the diffusive IV and the cumulative squared jumps:

$$QV = \int_0^1 \sigma_s^2 ds + \sum_{0 \leq s \leq N} J_s^2, \tag{6}$$

where J_s captures a jump (if present).

The RV Estimator is a consistent measure of the total QV in the presence of Jumps. Barndorff–Nielson and Shephard (2006) propose BPV (BiPower Variation) as a consistent estimator for the integrated variation. BPV is defined as:

$$BPV = \pi/2 \sum_{i=2}^{n} |r_i||r_{i-1}|. \tag{7}$$

The limit of BPV includes only the QV component related to the continuous element of the price process.

$$BPV \xrightarrow{p} \int_0^1 \sigma_s^2 ds. \tag{8}$$

Let us assume that $I_{t,\alpha}$ is a variable that takes the value 1 if a jump has been detected on day t at the α significance level (and 0 otherwise). Then the estimate of the realized jumps is given by:

$$J_{t,\alpha} = I_{t,\alpha}(RV_t - BPV_t). \tag{9}$$

1.3. RGARCH Model

Standard GARCH models use daily returns (generally squared returns) to extract information about the current level of volatility, and this information is used to form expectations about the next period's volatility. The implication is that GARCH models are poorly suited for situations where volatility is extreme.

In this section, we present the RGARCH model, whose main characteristic is conditional variance, $h_t = var(r_t|F_{t-1})$, where r_t is a return time series. In the GARCH(1,1) model, the conditional variance h_t is a function of h_{t-1} and r_{t-1}, whereas, in RGARCH, h_t will depend on x_{t-1}, which represents a RV measure, such as the realized variance. A measurement equation, which links the realized measurement to the hidden volatility, completes the model.

The RGARCH(1,1) model of Hansen et al (2012) is given by:

$$\begin{cases} r_t = \sqrt{h_t} z_t, \;\; z_t \sim i.i.d(0,1), \\ h_t = w + \gamma x_{t-1} + \beta h_{t-1}, \\ x_t = \xi + \phi h_t + \eta_1 z_t + \eta_2 (z_t^2 - 1) + \epsilon_t, \end{cases} \tag{10}$$

where we have defined the dynamics for the returns (r_t), the conditional variance (h_t) and the realized measure (x_t). To ensure that the long-run unconditional variance is finite and positive, the necessary conditions for the RGARCH(1,1) model are:

$$w + \gamma\xi > 0, 0 < \beta + \gamma\phi < 1,$$

It is sufficient that the parameters w, β, and γ be positive in order to guarantee the positivity of each h_t.

1.4. ANN

ANN have the advantage of approximating any nonlinear function (Cybenko, 1989). In this study, we use a multilayer perceptron (MLP) composed of an input layer, a hidden layer, and an output layer. The conventional back-propagation technique, which uses gradient descent, is used to minimize the quadratic error. The ANN with a single hidden layer used for forecasting is illustrated in Fig. 1.

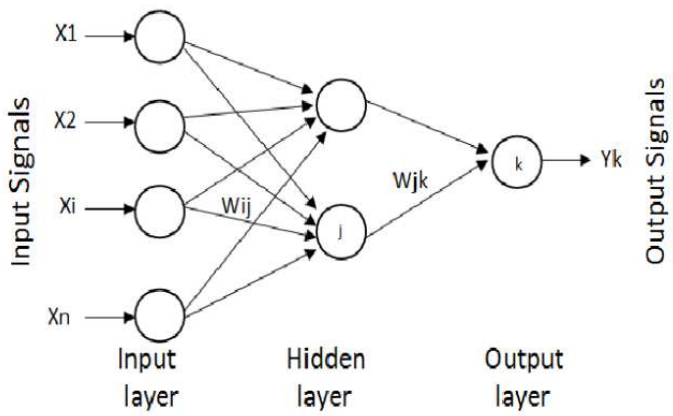


Fig. 1. The structure of FNN

Neurons in the input layer distribute input signals (x_i) to the neurons of the hidden layer. Each neuron j in the hidden layer receives all the other input signals (x_i) weighted with connection weights w_{ji}.

$$a_j = \sum_{i=1}^{n} w_{ji} x_i. \tag{11}$$

Each neuron j in the hidden layer computes its output as follows:

$$z_j = f(\sum_{i=1}^{n} w_{ji} x_i), \tag{12}$$

where f is an activation function.

The node of the output layer is defined as:

$$y_k = \sum_{j=1}^{m} w_{jk} f(\sum_{i=1}^{n} w_{ji} x_i). \tag{13}$$

FNN have a number of drawbacks, such as poor memory. It is impossible for FNN to remember prior inputs or states as they do not store any historical data and each input is processed independently. Therefore they are inappropriate for tasks requiring the capture of temporal dependencies or long-term memory. On the other hand, Recurrent neural networks (RNN) overcome these restrictions by integrating feedback connections, enabling them to record data from prior inputs.

1.5. Hybrid Models

In this paper, we look at how to incorporate RGARCH into an ANN structure. We expected that adding a neural network term to RGARCH would explain the delicate nonlinearity of *RV*. Even though RGARCH describes stylized *RV* facts very well, it is insufficient for capturing jumps in return volatilities. Many researchers suggested that continuous volatility and jump components have different dynamics and should thus be modeled separately. In terms of economics, identifying the jump component detects the risk associated with the jump. As a result, we can anticipate the links between political or economic news and price jumps. RGARCH ignores the possibility of a long memory in the return volatility series. The input variables of the ANN are chosen using the Granger causality test.

We proposed three neural network-based RGARCH models. To predict RV at time t, the first model is built by entering the forecast series of RV using RGARCH, $F(t)$, the previous Bipower variation $BPV(t-1)$, and a jump component $Jmp(t-1)$. The continuous and discontinuous components of QV have different dynamics, so, the ability to separate them may lead to improved predictions.

The second hybrid model is built by considering $F(t)$, the previous Bipower variation $BPV(t-1)$, $Jmp(t-1)$, $RV(t-1)$, the weekly RV $Vw(t-1)$, and the monthly RV $Vm(t-1)$ as input to the ANN, in order to capture the long memory of return volatility.

We also developed a third model where the same inputs as the second hybrid model are fed into a standard recurrent neural network (RNN) to benefit from its capacity to learn and retain information. RNN have recurrent connections that enable them to keep internal memory and capture temporal dependencies in the data, in contrast to FNN which receive inputs sequentially and one-way. This makes RNN especially suitable for situations where identifying underlying patterns or making correct predictions depends on the order and context of the input data. The performances of three neural network-based RGARCH models are evaluated using out-of-sample forecasting errors.

2. Data Analysis

We considered a simple realized measure estimator from the Oxford-Man Institute of Quantitative Finance (which contains daily information about stock indexes from 2000 to 2017) with a sampling frequency of the 5-min realized variance of the stock index for the period from 2012 to 2017.

We used correlation matrix graphs (Fig. 2) to identify three stock market indexes that exhibit low correlation among themselves. The data sample is subdivided into two sets, the first one contains 720 observations and is used to train and develop the model; the second set of 180 observations is used to test the model.

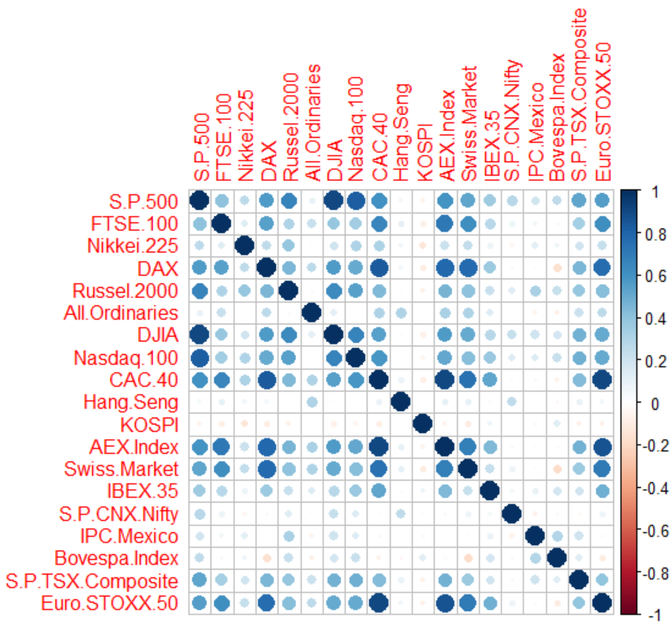


Fig. 2. Correlation matrix

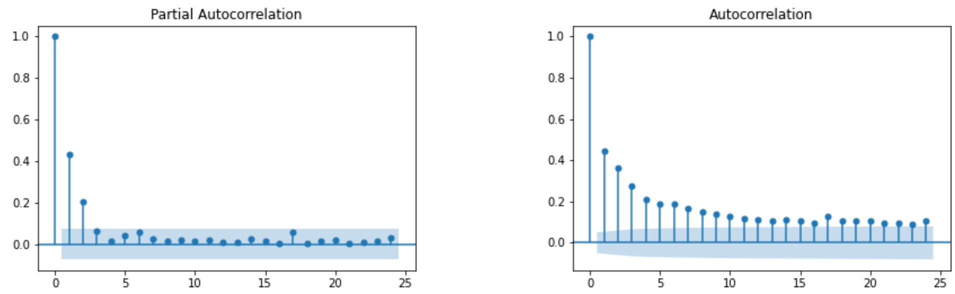
When examining the correlation matrix, we looked for values that were close to zero, which denotes a weak correlation between the variables. We were able to lower the risk of multi-collinearity and increase the quality of our analysis by identifying poorly correlated stock indices. This is particularly critical when developing predictive models or doing statistical studies.

Using correlation graphs allowed us to choose three stock market indices (Nasdaq100, IBEX 35, and All Ordinaries) with low correlation between them.

2.1. RGARCH

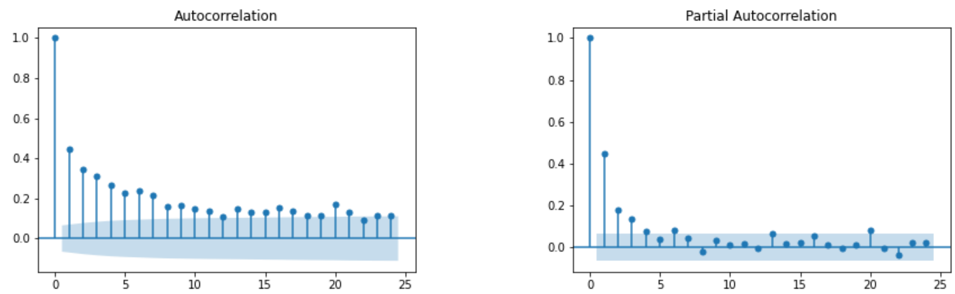
This study focuses on daily financial returns (r_t) of Nasdaq 100, IBEX 35, and All Ordinaries stock indexes multiplied by 100, from 01/03/2012 through 12/04/2017. The dataset is separated into two subsets. The training set represents 80% of the dataset, and the testing set represents 20%.

RGARCH is used to capture the mean features of volatility. The partial autocorrelation function (PACF) can be used for the identification of the autoregressive component of the RGARCH model. The correlograms of the three stock indexes' volatility are given in Figs. 3, 4, and 5.



(A) ACF with bounds for RV series (B) PACF with bounds for RV series

Fig. 3. ACF and PACF of Nasdaq 100 RV

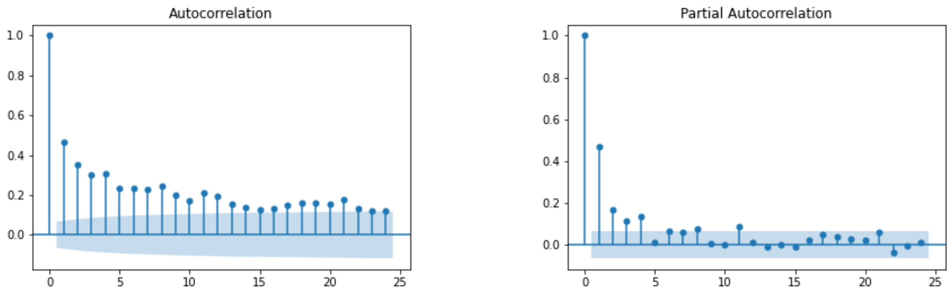


(A) ACF with bounds for RV series (B) PACF with bounds for RV series

Fig. 4. ACF and PACF of IBEX 35 RV

Table 1 shows the result of the Hurst exponent test, which is a statistical tool used to assess the long-term memory and predictability of time-series data, often known as the

Hurst coefficient. The exponent value varies from 0 to 1, with values above 0,5 indicating persistence while values below 0,5 suggest a random or uncorrelated series.



(A) ACF with bounds for RV series (B) PACF with bounds for RV series

Fig. 5. ACF and PACF of All Ordinaries RV

Table 1

Hurst exponent test

Stock Index	Nasdaq100	IBEX35	All Ordinaries
Empirical Hurst exponent	0,7717	0,6675	0,7558

All the results are above 0,5. This indicates that the data may have a memory that extends beyond short-term fluctuations. A high Hurst exponent indicates that price movements have a higher chance of continuing in the same direction. According to the correlogram and Table 1, the time series exhibits a slowly decreasing ACF over time, which indicates the presence of a long memory.

The correlogram shows that the Realized variance at time t presents a significant and positive correlation with its Realized variance at time $t-1$. We use the RGARCH model with the conditional variance, h_t, as a function of h_{t-1} and x_{t-1}. Table 2 indicates the parameter estimation of the RGARCH model.

The parameters of the RGARCH model demonstrate significance at the 5% level, with the exception of the ω parameter, which exhibits significance at the 13,8% level.

2.2. Hybrid Models

Data Preprocessing

In machine learning and AI, hybrid models have grown in significance. These models boost their performance and solve challenging real-world tasks by combining the advantages of different neural network architectures or by using other machine learning approaches. The advantages of several models are combined in the suggested hybrid model to increase accuracy, resilience, interpretability, and flexibility.

A critical step in determining the architecture of the neural networks is selection of variables. In this paper, we used the Granger causality test (Table 3) to optimally detect the relevant explanatory variables. After identifying the set of inputs, a suitable number of hidden layers and hidden neurons can be chosen.

According to the approximation theorems [20, 21], in theory, only one hidden layer suffices for the approximation of any sufficiently regular function. We utilized this rule of thumb to determine the number of neurons in the hidden layer.

Table 2

Parameters of the RGARCH model

Parameters	Coefficients	Std.errors	t-stats	P-Value
Nasdaq 100				
ω	0,065248	0,016619	3,92617	0,000086
γ	0,374533	0,031176	12,01333	0,000000
β	0,563362	0,028617	19,68595	0,000000
ξ	$-0,168856$	0,036386	$-4,64064$	0,000003
ϕ	1,066909	0,047236	22,58653	0,000000
η_1	$-0,108463$	0,011153	$-9,72489$	0,000000
η_2	0,095760	0,006349	15,08341	0,000000
IBEX 35				
ω	0,026176	0,017659	1,4823	0,138255
γ	0,322072	0,054362	5,9246	0,000000
β	0,656985	0,061559	10,6724	0,000000
ξ	$-0,067626$	0,040465	$-1,6712$	0,094682
ϕ	0,872304	0,057890	15,0683	0,000000
η_1	$-0,085037$	0,015258	$-5,5731$	0,000000
η_2	0,117116	0,009440	12,4065	0,000000
All Ordinaries				
ω	0,062612	0,021228	2,949465	0,003183
γ	0,189679	0,023508	8,068645	0,000000
β	0,752958	0,028289	26,617057	0,000000
ξ	$-0,477966$	0,057292	$-8,342585$	0,000000
ϕ	1,095063	0,075504	14,503374	0,000000
η_1	$-0,039646$	0,017302	$-2,291441$	0,021938
η_2	0,194495	0,009983	19,483439	0,000000

The activation function of the hidden layer is the sigmoid function that takes any real value as output values in the range of 0 to 1. It is differentiable and provides a smooth gradient. A gradient descent was used to train the FNN and RNN. We employed an early stopping method that allows the training to be halted when the validation loss no longer improves. This avoids overfitting and ensures that the network is trained for an optimal number of epochs.

The weights of neural networks are commonly initialized according to a law centered at zero with a standard deviation less than one. It would have been preferable to normalize the different features to accelerate the gradient descent's convergence. The results found after normalization show that the algorithm could never converge when training the model, which means that the normalization can lead to the loss of information.

The majority of our data are in the range 0 to 10, which makes finding an optimal result during gradient descent more difficult. The solution is to divide our data (inputs and target) by ten to generate data in the range 0 to 1. The results are presented in Table 3, where the probability is associated with the acceptance or rejection of the null hypothesis.

The five factors above have a significant causal link to the *RV*. Therefore, these factors are used as inputs in ANN. The graphic representation of the predicted results and forecasted volatility are given in Fig. 6, 7, 8.

Table 3

The results of the Granger causality test

Null hypothesis	Probability		
	Nasdaq 100	IBEX 35	All Ordinaries
F_{R_V} does not cause RV	0.0005378 ***	0.0001564 ***	4.131e-06 ***
Bipower variation does not cause RV	0.0351*	0.003403 **	0.006459 **
Jump component does not cause RV	0.0182 *	0.002858 **	0.05757
Weekly volatility does not cause RV	0.0108 *	0.03305 *	0.0015 **
Monthly volatility does not cause RV	0.0130 *	0.04156 *	0.0484 *

Signif : 0 ’***’ 0,001 ’**’ 0,01 ’*’ 0,05 ’.’ 0,1 ’ ’ 1

F_{RV}: represents the forecasted RV based on the RGARCH model.

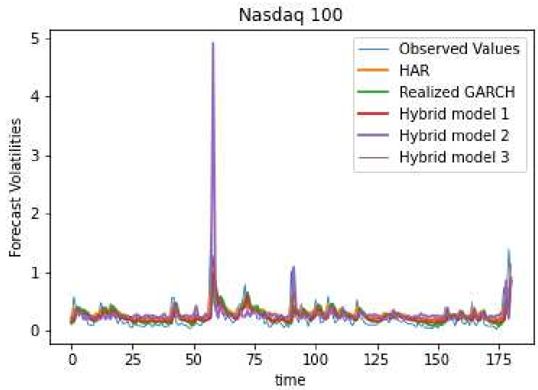


Fig. 6. Nasdaq 100 predicted testing results

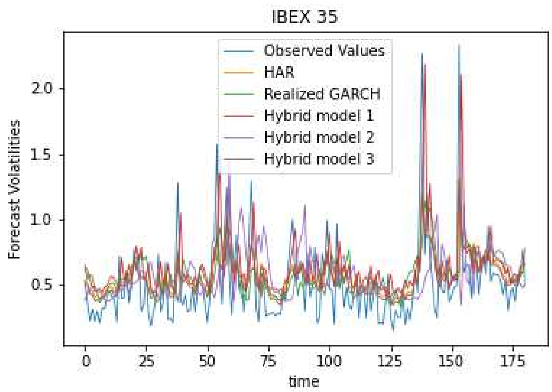


Fig. 7. IBEX 35 predicted testing results

Fig. 6, 7, and 8 show that the second hybrid model based on FNN exhibits poor performance and introduces noise into the predictive system, despite the fact that all of the input variables have a high correlation with the output.

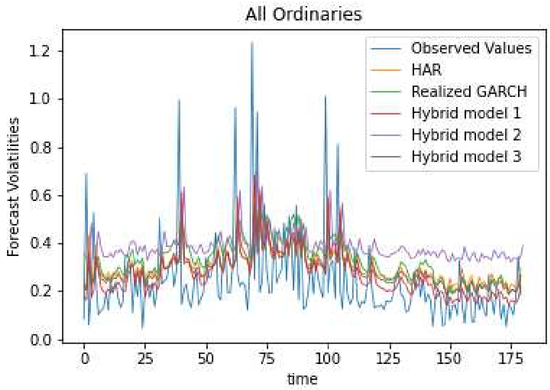


Fig. 8. All Ordinaries predicted testing results

2.3. A Comparison of Forecasting Performance Between Statistical Models and Hybrid Models

The comparison of volatility forecasts was conducted for a one-step ahead horizon in terms of mean squared error (MSE) defined as follows:

$$MSE = 1/N \sum_{i=1}^{N} (RV_{iobs} - RV_{ipred})^2,$$

where RV_{iobs} represents the observed value for $i = 1, 2, ..., N$. N is the number of out-of-sample observations, and RV_{ipred} is the predicted value by the model. 180 out-of-sample observations are used to forecast the volatilities and examine the performance of statistical and hybrid models.

2.3.1. Empirical Results

Figs. 9 and 10 provide the results of the statistical models and the proposed model to predict stock index volatilities using MSE and ρ.

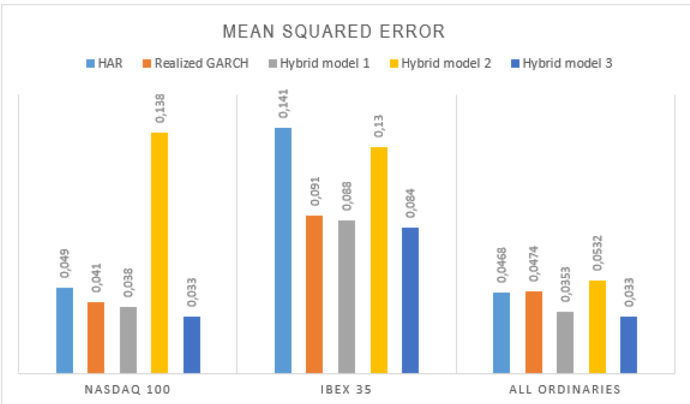


Fig. 9. The results of forecasting volatilities based on MSE

According to Figs. 9 and 10, the third hybrid model based on RNN performs better than model-based FNN and statistical models.

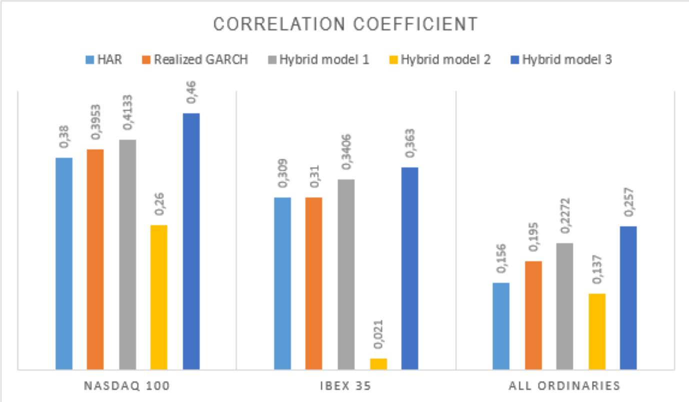


Fig. 10. The results of forecasting volatilities based on the correlation coefficient

Conclusion

Volatility forecasting plays a central role in financial applications such as asset allocation and option pricing, hedging, and risk analysis. Predicting stock index volatility is a very difficult task as the time series characterizing stock index movements are complex and difficult to model.

The GARCH family of models are the most widely used approach to modeling and forecasting asset return volatility. The GARCH model has been criticized for failing to account for asymmetric volatility. As a result, a new class of asymmetric GARCH specifications, such as RGARCH, has been introduced into the literature. This model takes into account the asymmetry of the effects due to the shocks, but it does not simulate the stock fluctuations with the jump effect well; and nor does it take long memory into consideration.

The proposed approach is based on a combination of the RGARCH and ANN models. The predicted value from the RGARCH model is the input variable and other factors that showed a causal link with the output variable using the Granger causality test.

The empirical results indicate that the stylized facts relating to the behavior of the volatility are better captured by the First Hybrid model. This means that the ANN model is improved by including the forecasted volatility based on the RGARCH model, the jump component, and the Bi-Power Variation of the stock index as input for the neural network. The findings also show that the addition of the RV of the previous day, the previous week, and the previous month to the FNN model generates noise in the predictive model.

FNN is sometimes referred to as a static network by researchers. Even when sample data display temporal dependency, FNN do not display memory. By permitting internal feedback, RNN overcome this limitation. For this reason, we employed RNN with a set of samples that includes all variables that exhibit a causal link with the output in order to simultaneously capture asymmetry, the jump effect, and long-term dependence. The results show that RNN outperformed FNN and classic econometric approaches.

In this paper, our focus has been on the statistical analysis of historical volatility. However, in practical applications, incorporating a combination of categories of information (such as economic, technical, and historical data) can enhance the accuracy of volatility predictions. This perspective highlights the potential for further research into the integration of diverse information sources for improved prediction outcomes.

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ПРОГНОЗИРОВАНИЕ ВОЛАТИЛЬНОСТИ ДОХОДНОСТИ АКЦИЙ С ИСПОЛЬЗОВАНИЕМ РЕАЛИЗОВАННОГО GARCH МОДЕЛЬ И ИСКУССТВЕННАЯ НЕЙРОННАЯ СЕТЬ

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Прогнозирование волатильности вызвало интерес ученых и практиков в области моделировании фондового рынка, распределения активов, ценообразования опционов и торговли на финансовых рынках. Это необходимо для управления рисками, распределения активов, ценообразования опционов и торговли на финансовых рынках. Это может быть сделано с помощью различных методов прогнозирования временных рядов и искусственных нейронных сетей (ИНС).

Текущее исследование посвящено моделированию и прогнозированию индекса фондового рынка с использованием высокочастотных данных. Недавнее исследование моделирования высокочастотной волатильности называется модель Realized-GARCH, где ключевой особенностью является уравнение измерения, которое связывает реализованную меру с условной дисперсией доходности. Затем, Realized-GARCH учитывает асимметрию эффектов, вызванных шоками.

В данной работе предлагается гибридная модель: ANN и модель Realized-GARCH для прогнозирования индекса волатильности доходности акций. Данная модель была создана путем введения прогнозируемой реализованной волатильности волатильности (RV) с использованием модели Realized GARCH в ИНС. Выбор входных переменных АНН был сделан с использованием теста причинности Грейнджера, чтобы уменьшить шум, который может повлиять на систему прогнозирования и который может быть порожден входной переменной переменной, не связанной статистически с поведением волатильности фондового рынка.

Результаты показывают, что гибридная модель превосходит модели Realized GARCH и HAR-типа во вневыборочной оценке по RMSE и коэффициенту корреляции.

Ключевые слова: волатильность; модель Realized-GARCH; гибрид; тест причинности Грейнджера.

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