MSC 35S10, 60G99 DOI: 10.14529/mmp240102

STABILITY OF SOLUTIONS TO THE STOCHASTIC OSKOLKOV EQUATION AND STABILIZATION

O.G. Kitaeva, South Ural State University, Chelyabinsk, Russian Federation, kitaevaog@sus.ru

This paper studies the stability of solutions to the stochastic Oskolkov equation describing a plane-parallel flow of a viscoelastic fluid. This is the equation we consider in the form of a stochastic semilinear Sobolev type equation. First, we consider the solvability of the stochastic Oskolkov equation by the stochastic phase space method. Secondly, we consider the stability of solutions to this equation. The necessary conditions for the existence of stable and unstable invariant manifolds of the stochastic Oskolkov equation are proved. When solving the stabilization problem, this equation is considered as a reduced stochastic system of equations. The stabilization problem is solved on the basis of the feedback principle; graphs of the solution before stabilization and after stabilization are shown.

Keywords: the Oskolkov equation; stochastic Sobolev-type equations; invariant manifolds; the stabilization problem.

Introduction

The Oskolkov equation

$$(\mathbb{I}-\chi\Delta)\Delta u_t=\nu\Delta^2 u-\frac{\partial(\psi,\Delta u)}{\partial(x_1,x_2)} \tag{1}$$

describes the plane-parallel flow of a viscoelastic liquid [1]. In [2], the initial boundary value problem for the equation in $D\times\mathbb{R}$ was studied, where $D\subset\mathbb{R}^2_{(x_1,x_2)}$ is a region with a smooth boundary. The smoothness and simplicity of the set, which is a phase space, is proved. In [3] studied the stability of solutions to equation (1).

We extend the results of [3] to the stochastic equation (1). To do this, we consider equation (1) as a stochastic semi-linear equation of the Sobolev type

$$L\overset{o}{\eta}=M\eta+N(\eta), \tag{2}$$

where, $\overset{o}{\eta}$ is the Nelson – Gliklikh derivative [4] of the stochastic process $\eta=\eta(t)$. The number of works on the study of stochastic equations of the Sobolev type is constantly growing. For example, in [5–7] study the solvability of a linear equation

$$L\overset{o}{\eta}=M\eta. \tag{3}$$

The papers [8, 9] consider the stochastic phase space of a nonlinear equation

$$L\overset{o}{\eta}=M(\eta). \tag{4}$$

The article [10] considers the stability of the stochastic linear Oskolkov equation

$$(\mathbb{I}-\chi\Delta)\Delta u_t=\nu\Delta^2 u. \tag{5}$$

Equation (5) is considered a linear stochastic equation (3). In the papers [11–13] find numerical stable and unstable solutions to equation (3). In the papers [14, 15] study the invariant manifolds of equation (2). The stabilization problem for equation (3) was considered and solved for the first time [16].

This article is a continuation of the work of [10] on the stability of the semilinear equation (1). Before proceeding to the study of the stability and instability of solutions to the stochastic Oskolkov equation, it is necessary to address the solvability of the stochastic equation (1). To do this, we extend the results of [1] to the stochastic Oskolkov equation. Then we apply the results of [15] to prove the existence of invariant spaces of the stochastic Oskolkov equation. To solve the problem of stabilizing unstable solutions to this equation, we extend the results of [16] to the semilinear equation (2).

The article contains three parts. The first part presents the results of the stable and unstable invariant manifolds of equation (2). The second part is devoted to the stabilization problem. In the third part, the results of the first and second parts are applied to the semilinear stochastic Oskolkov equation.

1. Invariant Manifolds of Stochastic Sobolev Type Equations

Let the space $\mathfrak{U}$ ($\mathfrak{F}$) be a separable Hilbert space, $\{\varphi_k\}$ ($\{\psi_k\}$) is the basis of the space $\mathfrak{U}$ ($\mathfrak{F}$), $\{\xi_k\}$ ($\{\zeta_k\}$) is a sequence of random variables from $\mathbf{L_2}$, and $\mathbf{K} = \{\lambda_k \subset \mathbb{R}_+\}$ is a sequence of real numbers, $\sum\limits_{k=1}^{\infty} \lambda_k^2 < \infty$. The space $\mathbf{U_K L}_2$ ($\mathbf{F_K L}_2$) is a replenishment of the linear shell of random variables of the form

$$\eta = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k \left(\zeta = \sum_{k=1}^{\infty} \lambda_k \zeta_k \psi_k \right)$$

according to the norm

$$\|\phi\|^2_{\mathbf{U_K L}_2} = \sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\xi_k, \quad \left(\|\zeta\|^2_{\mathbf{F_K L}_2} = \sum_{k=1}^{\infty} \lambda_k^2 \mathbf{D}\zeta_k \right).$$

Lemma 1. *The operator* $A \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$ *iff the same operator* $A \in \mathcal{L}(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$.

Let operators $L,\ M \in \mathcal{L}(\mathbf{U_K L}_2; \mathbf{F_K L}_2)$, $N \in C^\infty(\mathbf{U_K L}_2, \mathbf{F_K L}_2)$. Consider a semilinear stochastic equation of the Sobolev type (2) with an initial condition

$$\eta(0) = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k. \tag{6}$$

Stochastic $\mathbf{K}$-process $\eta \in \mathbf{C}^1(\mathcal{J}; \mathbf{U_K L}_2)$ we call the *solution* of equation (2) if probably almost all its trajectories satisfy equation (2) for all $t \in \mathcal{J}$, where $\mathcal{J} \subset \mathbb{R}$. The solution of $\eta = \eta(t)$ of equation (2) is called the *solution of the Cauchy problem* (2), (6) if the equality (6) holds for some random $\mathbf{L}$-value $\eta_0 \in \mathbf{U_L L}_2$.

Definition 1. [8] *The set* $\mathbf{P} \subset \mathbf{U_L L}_2$ *called a stochastic phase space of equation (2) if*
(i) probably almost every solution path $\eta = \eta(t)$ *of the equation (2) lies in* $\mathbf{P}$*;*

(ii) for almost all $\eta_0 \in \mathbf{P}$ exists a solution to the problem (2), (6).

If the operator M (L,p)-bounded operator, then operators

$$P = \frac{1}{2\pi i}\int\limits_{\gamma}(\mu L - M)^{-1}Ld\mu,\ Q = \frac{1}{2\pi i}\int\limits_{\gamma}L(\mu L - M)^{-1}d\mu$$

are projectors, where the contour γ bounds $\sigma^L(M)$. Denote $\mathbf{U}^1_{\mathbf{K}}\mathbf{L}_2 = \mathrm{im}P$. The subspace $\mathbf{U}^1_{\mathbf{K}}\mathbf{L}_2$ is the phase space of equation (3) [5].

Theorem 1. *Let the operator M (L,p)-bounded operator, operator $N \in C^1(\mathbf{U_K L}_2, \mathbf{F_K L}_2)$, and the set*

$$\mathbf{M} = \begin{cases} \{\eta \in \mathbf{U_K L}_2 : (\mathbb{I} - Q)(M\eta + N(\eta)) = 0\},\ \textit{if}\ \ker L \neq \{0\}; \\ \mathbf{U_K L}_2,\ \textit{if}\ \ker L = \{0\} \end{cases}$$

is a simple Banach C^1-manifold. Then the set $\mathbf{M}$ is the phase space of equation (2).

The proof of the theorem repeats the proof of a similar theorem in [8]. Therefore, it is not given here.

Let the following condition be fulfilled:

$$\left.\begin{array}{c} \sigma^L(M) = \sigma^L_r(M) \bigcup \sigma^L_l(M), \\ \sigma^L_{r(l)}(M) = \{\mu \in \sigma^L(M) : \mathrm{Re}\mu > 0\ (\mathrm{Re}\mu > 0)\} \end{array}\right\}. \tag{7}$$

There are projectors

$$P_{l(r)} = \frac{1}{2\pi i}\int\limits_{\gamma_{l(r)}}(\mu L - M)^{-1}Ld\mu.$$

Denote $\mathbf{I}^s = \mathrm{im}P_l$ and $\mathbf{I}^u = \mathrm{im}P_r$. If operator $M(L,p)$-bounded operator, then the spaces $\mathbf{I}^s$ and $\mathbf{I}^u$ are stable and unstable invariant spaces of equation (3) [10].

Consider the set

$$\mathbf{M}^s = \{\eta_0 \in \mathbf{U_K L}_2 : \|P_l\eta_0\|_{\mathbf{U_K L}_2} \leq R_1,\ \|\eta(t,\eta_0)\|_{\mathbf{U_L L}_2} \leq R_2,\ t \in \mathbb{R}_+\}$$

$$\Big(\mathbf{M}^u = \{\eta_0 \in \mathbf{U_K L}_2 : \|P_r\eta_0\|_{\mathbf{U_K L}_2} \leq R_1,\ \|\eta(t,\eta_0)\|_{\mathbf{U_L L}_2} \leq R_2,\ t \in \mathbb{R}_-\}\Big).$$

If the set $\mathbf{M}^s$ $(\mathbf{M}^u)$ is diffeomorphic to a closed ball in $\mathbf{I}^s$ $(\mathbf{I}^u)$, touches $\mathbf{I}^s$ $(\mathbf{I}^u)$ at the zero point, and $\|\eta(t,\eta_0)\|_{\mathbf{U_L L}_2} \to 0$ for any $\eta_0 \in \mathbf{M}^s$ $(\eta_0 \in \mathbf{M}^u)$ and for $t \to +\infty$ $(t \to -\infty)$, then the set $\mathbf{M}^s$ $(\mathbf{M}^u)$ is called the *stable(unstable) invariant manifold* of equation (2).

Theorem 2. [15] *Let the operator M (L,p)-bounded operator, condition (7), and the operator $N \in C^k(\mathfrak{U}, \mathfrak{F})$, $N(0) = 0$, $N'_0 = \mathbb{O}$. Then there are stable and unstable invariant manifolds of equation (2) in the neighborhood of the point zero.*

2. The Stabilization Problem

For stabilizing solutions, it is required to find such a control effect on equation (2) so that its solution $\eta = \eta(t)$ becomes stable, i.e.

$$\lim_{t\to+\infty}\|\eta(t)\|_{\mathbf{U_K L}_2} = 0. \tag{8}$$

Let condition (7) be fulfilled, then equation (2) will be considered as a reduced system

$$L_0 \overset{o}{\eta}{}^0 = M_0\eta^0 + N_0(\eta^0 + \eta^s + \eta^u), \tag{9}$$

$$L_s \overset{o}{\eta}{}^s = M_s\eta^s + N_s(\eta^0 + \eta^s + \eta^u), \tag{10}$$

$$L_u \overset{o}{\eta}{}^u = M_u\eta^u + N_u(\eta^0 + \eta^s + \eta^u), \tag{11}$$

where $F_0(\eta^0+\eta^s+\eta^u) = (\mathbb{I}-Q)N(\eta^0+\eta^s+\eta^u)$. The initial condition (6) is split as follows:

$$\eta_0^0 = (\mathbb{I} - Q)\eta_0 = \sum_{k=\{k:\ker L\neq\{0\}\}} \lambda_k\xi_k\varphi_k,$$

$$\eta_0^s = P_l P\eta_0 = P_l\eta_0 = \sum_{k=\{k:\mu_k\in\sigma_l^L(M)\}} \lambda_k\xi_k\varphi_k,$$

$$\eta_0^u = P_r P\eta_0 = P_r\eta_0 = \sum_{k=\{k:\mu_k\in\sigma_r^L(M)\}} \lambda_k\xi_k\varphi_k.$$

If the stochastic process $\eta = \eta(t)$ is the solution of equation (2), then this process satisfies system (9) – (11). Moreover, the solution of system (9) – (11) can be written as:

$$\eta = \eta^0(t) + \eta^s(t) + \eta^u(t), \tag{12}$$

where $\eta^0 = \eta^0(t)$ is the solution of equation (9), $\eta^s = \eta^s(t)$ is the solution of equation (10), $\eta^s = \eta^u(t)$ is the solution of equation (11).

Consider the following *stabilization problem*. Let us find such a stochastic process χ that condition (8) is satisfied for the solution of the system

$$L_0 \overset{o}{\eta}{}^0 = M_0\eta^0 + N_0(\eta^0 + \eta^s + \eta^u), \tag{13}$$

$$L_s \overset{o}{\eta}{}^s = M_s\eta^s + N_s(\eta^0 + \eta^s + \eta^u), \tag{14}$$

$$L_u \overset{o}{\eta}{}^u = M_u\eta^u + N_u(\eta^0 + \eta^s + \eta^u) + \chi. \tag{15}$$

Part of the relative spectrum of $\sigma^{L_s}(M_s)$ lies in the left half-plane of the complex plane. From theorem 2 it follows that

$$\lim_{t\to+\infty} \|\eta^s(t)\|_{\mathbf{U_K L_2}} = 0. \tag{16}$$

The stochastic process of χ is determined using feedback

$$\chi = B\eta^u. \tag{17}$$

Let us find an operator B such that the spectrum $\sigma^{L_u}(M_u + B)$ lies in the left half-plane of the complex plane. Denote $m = \max\limits_{\mu\in\sigma_u^L(M)} \{\mathrm{Re}\mu : \mathrm{Re}\mu > 0\}$. Let operator B have the form

$$B = -(\varepsilon + m)\mathbb{I}, \tag{18}$$

where $\varepsilon > 0$ can be arbitrarily small. Then the relative spectrum $\sigma^{L_u}(M_u + B)$ lies to the left of the imaginary axis. From theorem 2 it follows that

$$\lim_{t\to+\infty} \|\eta^u(t)\|_{\mathbf{U_K L_2}} = 0. \tag{19}$$

The solution $\eta = \eta(t)$ of system (13) – (15) can be written as

$$\eta = (\mathbb{I} + \delta)(\eta^1) = (\mathbb{I} + \delta)(\eta^s + \eta^u),$$

where the diffeomorphism $\delta = P^{-1}$. Then, for system (13)–(15) with feedback(17), by virtue of equalities (16) and (19), equality (8) will be fulfilled.

3. Invariant Manifolds of the Stochastic Oskolkov Equation

Let

$$\mathfrak{U} = \{u \in W_2^4 : u(x_1, x_2) = \Delta u(x_1, x_2) = 0,\ (x_1, x_2) \in \partial\Omega\},\ \mathfrak{F} = L_2(D).$$

The basis in space $\mathfrak{U}$ ($\mathfrak{F}$) is a family of eigenfunctions $\{\varphi_k\}$ of the Laplace operator Δ, orthonormal with respect to the scalar product $\langle\cdot,\cdot\rangle$ ($[\cdot,\cdot]$) in $\mathfrak{U}$ ($\mathfrak{F}$). Define the operators

$$L : \eta \to (\lambda - \Delta)\Delta\eta,\ M : \eta \to \nu\Delta^2\eta,\ N : \eta \to -\frac{\partial(\eta, \Delta\eta)}{\partial(x_1, x_2)}.$$

Operators $L,\ M \in \mathcal{L}(\mathbf{U_K L}_2, \mathbf{F_K L}_2)$.

Lemma 2. *(i) The operator M is $(L, 0)$-bounded;*
(ii) the operator $N \in C^\infty(\mathbf{U_K L}_2, \mathbf{F_K L}_2)$, $N(0) = 0,\ N'_0 \equiv \mathbb{O}$.

Proof. (i) The relative spectrum of the operator M has the form

$$\sigma^L(M) = \left\{\frac{\nu\nu_k}{1 - \chi\nu_k} : k \in \mathbb{N} \setminus \{l : \chi^{-1} = \nu_l\}\right\},$$

and is obviously limited. Here $\{\nu_k\}$ is the spectrum of the operator Δ. Then the operator M (L, σ)-bounded. The kernel of operator L has the form

$$\ker L = \begin{cases} \{0\},\ \chi^{-1} \neq \nu_k,\ k \in \mathbb{N}; \\ \{\varphi_k\},\ \chi^{-1} = \nu_k. \end{cases}$$

Let $\varphi \in \ker L\backslash\{0\}$, i.e.

$$\varphi = \sum_{\chi^{-1}=\nu_l} a_l\nu_l,\ \sum_{\chi^{-1}=\nu_l} |a_l| > 0,$$

then $\varphi \in \ker L \setminus \{0\}$,

$$\varphi = \sum_{\chi^{-1}=\nu_l} a_l\varphi_l,\ \sum_{\chi^{-1}=\nu_l} |a_l| > 0,$$

and

$$M\varphi = \nu\chi^{-1} \sum_{\chi^{-1}=\nu_k} a_k\varphi_k \notin \mathrm{im}L.$$

(ii) Let us calculate the Frechet derivatives

$$N'_\eta = -\frac{\partial(\eta, \Delta\cdot)}{\partial(x_1, x_2)} - \frac{\partial(\cdot, \Delta\eta)}{\partial(x_1, x_2)},\ N''_\eta = -\frac{\partial(\cdot, \Delta\cdot)}{\partial(x_1, x_2)} - \frac{\partial(\cdot, \Delta\cdot)}{\partial(x_1, x_2)}.$$

All other derivatives are zero. Note, that $N'_\eta,\ N''_\eta \in \mathcal{L}(\mathbf{U_K L}_2, \mathbf{F_K L}_2)$, and $N(0) = 0$, $N'_0 \equiv \mathbb{O}$.

□

Consider the set $\mathbf{M}$ and the space $\mathbf{U^1_K L}_2$. They have the following form:

$$\mathbf{M} = \begin{cases} \mathbf{U_K L}_2,\ \chi^{-1} \notin \{\nu_k\}; \\ \{\eta \in \mathbf{U_K L}_2 : \langle M\eta + N(\eta), \varphi_l\rangle = 0,\ \chi^{-1} = \nu_l\}, \end{cases}$$

$$\mathbf{U^1_K L}_2 = \begin{cases} \mathbf{U_K L}_2,\ \chi^{-1} \notin \{\nu_k\}; \\ \{\eta \in \mathbf{U_K L}_2 : \langle \eta, \varphi_l\rangle = 0,\ \chi^{-1} = \nu_l\}. \end{cases}$$

Theorem 3. *Let $\lambda \in \mathbb{R},\ \nu \in \mathbb{R}\setminus\{0\}$. Then the set $\mathbf{M}$*
(i) is a simple Banach C^∞-manifold;
(ii) is the phase space of stochastic equation (1).

Let $\nu > 0$, then $\sigma(M)^L = \sigma(M)^L_1 \bigcup \sigma(M)^L_2$, where

$$\sigma^L_1(M) = \left\{\frac{\nu\nu_k}{1-\chi\nu_k} :\ \nu_k > \chi^{-1}\right\},\ \sigma^L_2(M) = \left\{\frac{\nu\nu_k}{1-\chi\nu_k} :\ \nu_k < \chi^{-1}\right\}.$$

Consider the spaces

$$\mathbf{I}^u = \{\eta \in \mathbf{U_K L}_2 :\ \langle \eta, \varphi_k\rangle\varphi_k = 0,\ \nu_k > \chi^{-1}\},$$

$$\mathbf{I}^s = \{\eta \in \mathbf{U_K L}_2 : \langle \eta, \varphi_k\rangle\varphi_k = 0,\ \nu_k < \chi^{-1}\},$$

and consider the invariant sets

$$\mathbf{M}^u = \{\eta \in \mathbf{M} :\ \eta = \sum_{k:\ \nu_k > \chi^{-1}} \lambda_k\xi_k\varphi_k\},\ \mathbf{M}^s = \{\eta \in \mathbf{M} :\ \eta = \sum_{k:\ \nu_k < \chi^{-1}} \lambda_k\xi_k\varphi_k\}.$$

Theorem 4. *Let $\chi \in \mathbb{R}_-,\ \nu \in \mathbb{R}_+$. Then the set $\mathbf{M}^u$ is a finite-dimensional unstable invariant manifold of stochastic equation (1), and the set $\mathbf{M}^s$ is an infinite-dimensional stable invariant manifold of stochastic equation (1).*

Let $\chi \in \mathbb{R}_-,\ \nu \in \mathbb{R}_+$. Next, we consider the stabilization problem. For the random process χ, we search in the form $\chi = B\vartheta^u$. The B operator is defined by: $B = -\nu(\varepsilon + m)$, where $m = \max\limits_{\mu_k = \frac{\nu\nu_k}{1-\chi\nu_k}} \{\mathrm{Re}\mu_k,\ \chi^{-1} > \nu_l\}$, and ε we can be as small as we want. Then the relative spectrum

$$\sigma^{L_u}(M_u + B) = \left\{\frac{\nu\nu_k - \nu(\varepsilon + m)}{1-\chi\nu_k}\right\}$$

lies in the left half-plane of the complex plane.

Let the parameters $\chi = -0,1,\ \nu = 0,1$. Fig. 1 shows a graph of the solution of the stochastic Oskolkov equation before stabilization, and Fig. 2 shows a graph of the solution after stabilization for $\xi_1 = 0,2353939952,\ \xi_2 = -1,079196234,\ \xi_3 = -0,1231141571,$ $\xi_4 = 1,625165927$ in section $x_2 = 1$.

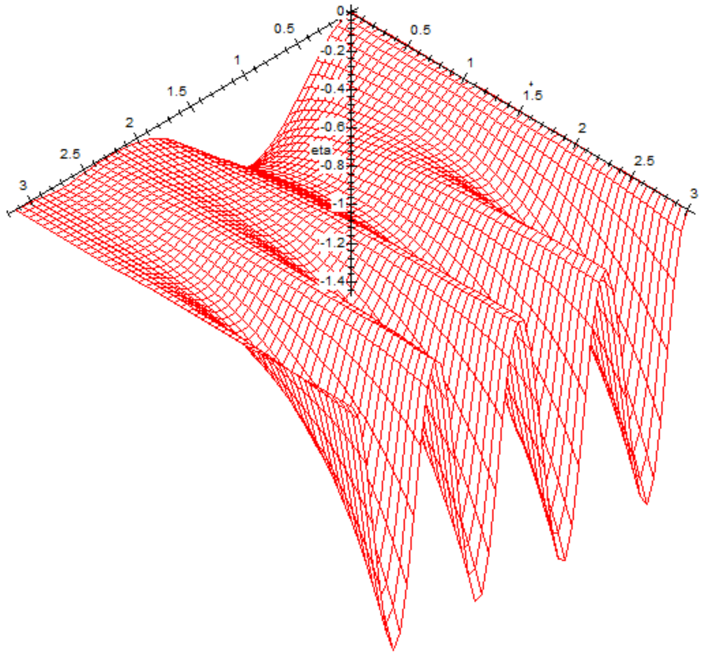

Fig. 1. Graph of the solution of the stochastic equation (1) before stabilization for $\chi = -0,1$, $\nu = 0,1$

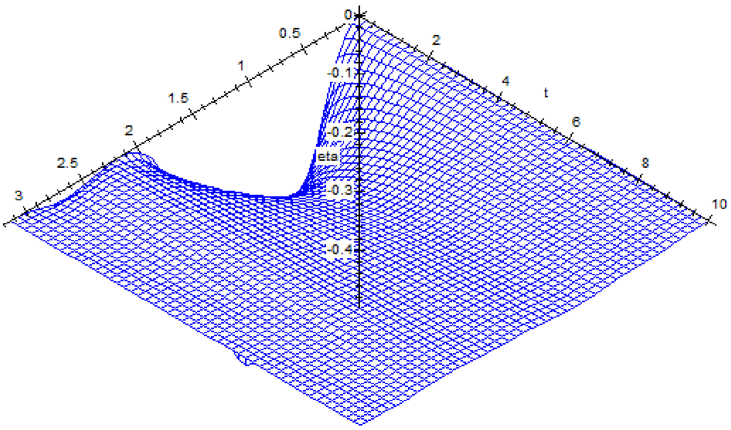

Fig. 2. Graph of the solution of the stochastic equation (1) after stabilization for $\chi = -0,1$, $\nu = 0,1$

Conclusion

In the future, we propose studying the stability of equations of form (2) in a relatively sectorial case. We also propose finding a method for solving the stabilization problem in the case when the relative spectrum intersects with the imaginary axis.

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Received August 3, 2023

УДК 517.9 **DOI: 10.14529/mmp240102**

УСТОЙЧИВОСТЬ РЕШЕНИЙ СТОХАСТИЧЕСКОГО УРАВНЕНИЯ ОСКОЛКОВА И СТАБИЛИЗАЦИЯ

О.Г. Китаева, Южно-Уральский государственный университет, г. Челябинск, Российская Федерация

В данной работе исследуется устойчивость решений стохастического уравнения Осколкова, описывающего плоскопараллельное течение вязкоупругой жидкости. Это уравнение мы рассматриваем в виде стохастического полулинейного уравнения соболевского типа. Во-первых, мы рассмотрим разрешимость стохастического уравнения Осколкова методом стохастического фазового пространства. Во-вторых, мы рассмотрим устойчивость решений этого уравнения. Доказаны необходимые условия существования устойчивых и неустойчивых инвариантных многообразий стохастического уравнения Осколкова. При решении задачи стабилизации это уравнение рассматривается как редуцированная стохастическая система уравнений. Задача стабилизации решается на основе принципа обратной связи; показаны графики решения до стабилизации и после стабилизации.

Ключевые слова: производная Нельсона – Гликлиха; стохастические уравнения соболевского типа; инвариантные многообразия, задача стабилизации.

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Ольга Геннадьевна Китаева, кандидат физико-математических наук, доцент, доцент кафедры «Уравнения математической физики», Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), kitaevaog@susu.ru.

Поступила в редакцию 3 августа 2023 г.