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INFLUENCE TO NEW FORMULAS GRADIENT FOR REMOVING IMPULSE NOISE IMAGES

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In conjugate gradient techniques, the conjugate formula is often the primary point of concentration. The conjugate gradient technique is used to solve problems that arise during the process of picture restoration. By using the quadratic model, a brand-new coefficient conjugate will be produced for the operation. The algorithms demonstrate both local and global convergence and descent. The numerical testing revealed that the newly developed method is much superior to the one that came before it. The recently created conjugate gradient strategy has better performance than the FR conjugate gradient technique, which is the industry standard.

Keywords: influence to formula gradient; convergence property; impulse noise reduction for images.

Introduction

One of the most fundamental difficulties in image processing is creating an image declaration [1] from noisy input data. The following phases may be used to breakdown the two-phase approach: AMF of the median type is used in the first step in order to precisely pinpoint the location of the impulse noise [2].

The approach that consists of two phases only recently saw publication in [3]. The adaptive center-weighted median filter (ACWMF) [4] is used during the first phase, which involves the identification of noise pixels by the use of the adaptive median filter (AMF) [5] for random-valued noise. Let x denote the original image with $M-$by$-N$ pixels and $\mathrm{A} = \{1, 2, 3, \ldots, M\} \times \{1, 2, 3, \ldots, N\}$ be the index set of x. Let the set of indices of the noisy pixels detected in the first phase denote by T, where $T \subset \mathrm{A}$ These were identified during the first stage. In the second stage, the recovered noise pixels are then used to minimize the following functional:

$$f_\alpha(u) = \sum_{(i,j)\in \mathrm{T}} \left[|u_{i,j} - y_{i,j}| + \frac{\beta}{2} \left(S^1_{i,j} + S^2_{i,j} \right) \right]. \tag{1}$$

Regularization parameters β and are also included

$$S^1_{i,j} = 2 \sum_{(m,n)\in \mathrm{P}_{i,j}\cap \mathbb{T}} \varphi_q \left(u_{i,j} - y_{m,n} \right),$$

$$S^2_{i,j} = \sum_{(m,n)\in P_{i,j}/T} \varphi_\alpha \left(u_{i,j} - y_{m,n} \right),$$

see [6]. The φ_α is an edge-preserving function where $\varphi_\alpha = \sqrt{\alpha + u^2}$, and $u = \lfloor u_{i,j} \rfloor_{(i,j)\in \mathrm{T}}$ be a column vector of length c ordered lexicographically (c is the number of elements of T), and $y_{i,j}$ denote the observed pixel value at position (i, j). Noisy pixels are restored without smoothing:

$$f_\alpha(u) = \sum_{(i,j)\in \mathrm{T}} \left[2.S^1_{i,j} + S^2_{i,j}\right]. \tag{2}$$

Optimization issues may be solved using nonlinear conjugate gradient algorithms due to their cheap memory utilization and simple iteration:

$$\mathrm{Min}\, f_\alpha(u), u \in \mathrm{R}^{|T|}, \tag{3}$$

where f is continuously differentiable, see [7]. This article discusses the nonlinear conjugate gradient technique, which is often used to solve (4) problems of the following type:

$$x_{k+1} = x_k + \alpha_k d_k, \tag{4}$$

where α_k is a step length and the search direction d_{k+1} are generated as:

$$d_{k+1} = -g_{k+1} + \beta_k d_k, \tag{5}$$

where β_k denotes the conjugate coefficient of the expression (see [3,8]). Global convergence features of conjugate gradient methods are very interesting. The Hestenes–Stiefel (HS) technique was one of the most efficient CG methods, but it failed to fulfill global convergence criteria under conventional line search [9]. The Fletcher and Reeves (FR) approach had the best convergence results [10]. Methods include:

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}, \quad \beta_k^{HS} = \frac{y_k^T g_{k+1}}{d_k^T y_k}, \tag{6}$$

where $y_k = g_{k+1} - g_k$. See [9,11–14] for good research on contemporary CG approaches with notable results. The conjugacy criteria makes the Hestenes–Stiefel formula acceptable. The goal of these strategies is to speed up the convergence of the Newton method. According to the idea of the quasi-Newton direction, the d_{k+1} in equation (5) comes close to representing the quasi-Newton method. Thus, a parameter for the β_k that:

$$-Q_{k+1}^{-1} g_{k+1} = -g_{k+1} + \beta_k s_k, \tag{7}$$

where Q_{k+1} is a Hessian matrix, see Nazareth [15]. New research seeks a globally convergent descent-conforming search direction to maximize conjugate gradient benefits. Numerous novel optimization approaches exist. Theoretical and mathematically, idea-rich techniques like [5] are effective. Numerical results show that the innovation technique [3, 4, 6] is more effective than the optimization approach [16]. In [17], the step size is estimated using a variety of line search techniques, including precise line search, as seen below:

$$\alpha_k = -\frac{g_k^T d_k}{d_k^T G d_k}. \tag{8}$$

We often use the strong Wolfe–Powell (SWP) [3] and [18] line search to determine the step length. The definition of SWP line search is as follows:

$$\begin{gathered} f\left(x_k + \alpha_k d_k\right) \le f\left(x_k\right) + \delta \alpha_k g_k^T d_k, \\ d_k^T g\left(x_k + \alpha_k d_k\right) \ge \sigma d_k^T g_k, \end{gathered} \tag{9}$$

where $0 < \delta < \sigma < 1$. In order to get more details, please refer to [1] and [19]. If you want more details on the convergence analysis that our approach generates, please have a look at reference [20].

As a reaction to this, we recommend to look into making some more tweaks, which, by our assumption, can improve the numerical performance. These alterations are predicated on the formulation of a new conjugate gradient parameter and the examination of the quadratic model's ability to analyze the convergence of that parameter's value.

1. Deriving the New Parameter

The idea of the research is to derive the new formulas. Using the Taylor formula for the objective function f(x), we have:

$$f(u) = f\left(u_{k+1}\right) + g_{k+1}^T\left(u - u_{k+1}\right) + \frac{1}{2}\left(u - u_{k+1}\right)^T Q\left(u_k\right)\left(u - u_{k+1}\right), \tag{10}$$

where the gradient is provided by:

$$g_{k+1} = g_k + Q\left(u_{k+1}\right) s_k. \tag{11}$$

It is possible to obtain the second-order curvature from (10) and (11):

$$s_k^T Q\left(u_k\right) s_k = \left(f_{k+1} - f_k\right) - 2/3 s_k^T g_k. \tag{12}$$

Using some algebra, we can derive:

$$s_k^T Q\left(u_k\right) s_k = 2/3 s_k^T y_k + 2/3\left(f_k - f_{k+1}\right). \tag{13}$$

The matrix $Q\left(u_k\right)$ answer is:

$$Q\left(u_k\right) = \frac{2/3 s_k^T y_k + 2/3\left(f_k - f_{k+1}\right)}{s_k^T s_k} I_n, \tag{14}$$

where I_n is the identity matrix. If we swap (14) for (7), we obtain:

$$\beta_k = \left(1 - \frac{s_k^T s_k}{2/3 s_k^T y_k + 2/3\left(f_k - f_{k+1}\right)}\right)\frac{g_{k+1}^T y_k}{s_k^T y_k}. \tag{15}$$

By using the formula shown above, we are able to write:

$$\beta_k^{\mathrm{BN1}} = \frac{1}{s_k^T y_k}\left(y_k - \frac{s_k^T y_k}{2/3 s_k^T y_k + 2/3\left(f_k - f_{k+1}\right)} s_k\right)^T g_{k+1}. \tag{16}$$

Using exact line search on equation (12), equation (16) yields:

$$\beta_k^{\mathrm{BN2}} = \frac{1}{s_k^T y_k}\left(y_k - \frac{s_k^T y_k}{2/3\left(f_k - f_{k+1}\right) - 2/3 s_k^T g_k} s_k\right)^T g_{k+1} \tag{17}$$

and

$$\beta_k^{\mathrm{BN3}} = \frac{1}{s_k^T y_k}\left(y_k - \frac{s_k^T y_k}{2/3 \alpha_k g_k^T g_k + 2/3\left(f_k - f_{k+1}\right)} s_k\right)^T g_{k+1}. \tag{18}$$

For convenience, these approaches are BN1, BN2, and BN3. New algorithm BN is given.

Algorithm BN

Initialization. Given $x_0 \in R^n$, set $k = 0, d_0 = -g_0$.

Stage 1 : If $\|g_k\| \leq \varepsilon$ then stop.

Stage 2 : Find α_k by (8) and (9).

Stage 3 : Let $x_{k+1} = x_k + \alpha_k d_k$, and compute β_k by (16-18).

Stage 4 : Compute $d_{k+1} = -g_{k+1} + \beta_k d_k$.

Stage 5 : Set $k = k + 1$ and go to stage 2.

2. Convergence Analysis for Uniformly Convex Function

Examining the global convergence of business intelligence algorithms is the next item on the agenda. Some of the presumptions we make are as follows.

Assumptions:

(i) $f(x)$ is bounded below on R^n and bounded on the set $\Psi = \{x \in R^n : f(x) \leq f(x_0)\}$.

(ii) g is Lipschitz continuous, i.e. there exists a nonnegative steady L such that:

$$\|g(u) - g(w)\| \leq L\|u - w\|, \quad \forall u, w \in R^n. \tag{19}$$

There is a constant $\Gamma \geq 0$, which means that $\|\nabla f(x)\| \leq \Gamma$, despite the fact that certain function assumptions have been made. You may get additional information about this topic in [7].

It is possible to demonstrate that the descent condition in the following lemma is rather helpful.

Theorem 1. *Only $s_k^T y_k \neq 0$ and the search directions from (6) and (14) make d_{k+1} a descent direction.*

Proof. Since $d_0 = -g_0$, we have $g_0^T d_0 = -\|g_0\|^2 < 0$. Assume that $d_k^T g_k \leq 0$ is true. Multiplying (5) by g_{k+1}, to get:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \left(1 - \frac{s_k^T s_k}{2/3 s_k^T y_k + 2/3\left(f_k - f_{k+1}\right)}\right) \frac{g_{k+1}^T y_k}{s_k^T y_k} s_k^T g_{k+1}. \tag{20}$$

Therefore,

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \left(1 - \frac{s_k^T s_k}{s_k^T y_k}\right) \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k^T g_{k+1}. \tag{21}$$

By Lipschitz yields $y_k^T g_{k+1} \leq L s_k^T g_{k+1}$ and $s_k^T y_k \leq L s_k^T s_k$. This lets us write:

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + \left(\frac{L s_k^T s_k - s_k^T s_k}{s_k^T y_k}\right) \frac{L\left(s_k^T g_{k+1}\right)^2}{s_k^T y_k}. \tag{22}$$

However, since L-1 is small zero, observe that:

$$d_{k+1}^T g_{k+1} \leq 0. \tag{23}$$

Theorem might be proven.

□

Dai et al. [21] illustrate the general conclusion in the following way for any conjugate gradient method using the Wolfe line search.

Lemma 1. *Consider any conjugate gradient method* (2) *with* $d_{k+1} = -g_{k+1} + \beta_k d_k$*, as a descent direction and* α_k *chosen by the strong Wolfe line search if assumptions (i), (ii) are true. If*

$$\sum_{k>1} \frac{1}{\|d_{k+1}\|^2} = \infty. \tag{24}$$

Then

$$\lim_{k\to\infty} (\inf \|g_{k+1}\|) = 0. \tag{25}$$

As a result, we may prove the following conclusion using Lemma 1, see [4] and [21]. We may demonstrate the following outcome using the lemma 1 condition.

Theorem 2. *If a constant* $\mu > 0$ *exists that, for every* k*, satisfies the following criteria:*

$$(\nabla f(u) - \nabla f(w))^T \geq \mu \|u - w\|^2, \forall u, w \in R^n. \tag{26}$$

Assuming the conditions of Lemma 1, we have

$$\lim_{k\to\infty} (\inf \|g_{k+1}\|) = 0. \tag{27}$$

Proof. It is evident from (11) that

$$\|d_{k+1}\| = \|g_{k+1}\| + \left|(1-\omega)\frac{g_{k+1}^T y_k}{s_k^T y_k}\right| \|s_k\|, \tag{28}$$

where $\omega = s_k^T s_k / 2/3 s_k^T y_k + 2/3\,(f_k - f_{k+1})$. It is clear from using Cauchy's inequality that:

$$\begin{aligned} \|d_{k+1}\| &\leq \|g_{k+1}\| + |(1-\omega)|\frac{\|g_{k+1}\|\,\|y_k\|}{\|s_k\|\,\|y_k\|}\,\|s_k\| \leq \\ &\leq (2-\omega)\,\|g_{k+1}\|. \end{aligned} \tag{29}$$

Thus, $\|\nabla f(x)\| \leq \Gamma$ implies that:

$$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} \geq \left(\frac{1}{2-\omega}\right)\frac{1}{\Gamma}\sum_{k\geq 1} 1 = \infty \tag{30}$$

by applying Lemma 1, implies that $\liminf_{k\to\infty} \|g_k\| = 0$.

□

3. Numerical Results

The effectiveness of the BN1, BN2, and BN3 algorithms for removing salt-and-pepper impulse noise is shown via some numerical data (3). The following are the parameters for the BN1, BN2, and BN3 procedures and Table 1 shows the investigation's flow chart. The first test photos may be seen in Table 1. All simulations are run using MATLAB 2015a on a PC. We contrast the BN1, BN2, and BN3 strategies to the FR method to see how well they perform, see [22–25]. Keep in mind that in this study, our primary concern is how rapidly we can resolve the minimization problem (3). The PSNR, or signal-to-noise ratio, is a statistic used to assess the effectiveness of recovered pictures:

$$PSNR = 10\log_{10}\frac{255^2}{\frac{1}{MN}\sum_{i,j}\left(u^r_{i,j}-u^*_{i,j}\right)^2}, \tag{31}$$

where the pixel values of the restored picture are marked by $u^r_{i,j}$ and the values of the original image are denoted by $u^*_{i,j}$. the original image's values were lost during the restoration process. The following are some of the circumstances that will result in the ineffectiveness of either approach:

$$\frac{|f(u_k)-f(u_{k-1})|}{|f(u_k)|} \le 10^{-4} \text{ and } \|f(u_k)\| \le 10^{-4}(1+|f(u_k)|). \tag{32}$$

The results of the tests are shown in Table 1 below. The report also includes the highest signal-to-noise ratio, the number of function evaluations, and the overall number of iterations (PSNR).

Tables 1 – 3 show that FR approaches BN1, BN2, and BN3 are quickest with the fewest iterations and function evaluations. All three strategies provide high PSNR. Figures 1, 2 and 3 show FR, BI1, BI2, and BI3 restoration results. These findings suggest that BN1, BN2, and BN3 may repair damaged photographs.

Table 1

Numerical results of FR and New BN1 algorithm

Image	Noise	FR-Method			BN1-Method		
	level r (%)	NI	**NF**	PSNR (dB)	**NI**	NF	PSNR (dB)
Le	50	82	153	30,5529	58,0	109,0	30,3946
	70	81	155	27,4824	56,0	104,0	27,4182
	90	108	211	22,8583	55,0	101,0	22,7628
Ho	50	52	53	30,6845	39,0	72,0	34,7204
	70	63	116	31,2564	44,0	79,0	31,1129
	90	111	214	25,287	53,0	95,0	25,0078
El	50	35	36	33,9129	30,0	53,0	33,8753
	70	38	39	31,864	34,0	60,0	31,7971
	90	65	114	28,2019	43,0	81,0	28,1453
c512	50	59	87	35,5359	35,0	68,0	35,3054
	70	78	142	30,6259	40,0	79,0	30,6412
	90	121	236	24,3962	51,0	101,0	24,8992

The provided methods outperform the FR method in terms of peak signal to noise ratio, iterations, and function evaluations.

Conclusions

In conclusion, we provided a new conjugate gradient formula that had been changed and presented the BN1, BN2, and BN3 conjugate gradient procedures. In addition, we

Table 2

Numerical results of FR and New BN2 algorithms

Image	Noise	FR-Method			BN2-Method		
	level r (%)	NI	**NF**	PSNR (dB)	NI	NF	PSNR (dB)
Le	50	82	153	30,5529	55,0	111,0	30,3861
	70	81	155	27,4824	54,0	108,0	27,3853
	90	108	211	22,8583	53,0	105,0	22,8229
Ho	50	52	53	30,6845	35,0	70,0	34,9326
	70	63	116	31,2564	43,0	84,0	31,0397
	90	111	214	25,287	57,0	111,0	24,8616
El	50	35	36	33,9129	27,0	52,0	33,8617
	70	38	39	31,864	31,0	60,0	31,7785
	90	65	114	28,2019	39,0	75,0	27,8699
c512	50	59	87	35,5359	37,0	74,0	35,4821
	70	78	142	30,6259	40,0	81,0	30,8268
	90	121	236	24,3962	53,0	107,0	25,0205

Table 3

Numerical results of FR and New BN3 algorithms

Image	Noise	FR-Method			BN3-Method		
	level r (%)	NI	**NF**	PSNR (dB)	NI	NF	PSNR (dB)
Le	50	82	153	30,5529	55,0	107,0	30,6604
	70	81	155	27,4824	58,0	113,0	27,3285
	90	108	211	22,8583	51,0	102,0	22,8401
Ho	50	52	53	30,6845	34,0	68,0	34,6449
	70	63	116	31,2564	42,0	83,0	31,0143
	90	111	214	25,287	56,0	108,0	25,0585
El	50	35	36	33,9129	29,0	55,0	33,8908
	70	38	39	31,864	34,0	65,0	31,9092
	90	65	114	28,2019	44,0	85,0	28,3056
c512	50	59	87	35,5359	33,0	66,0	35,3312
	70	78	142	30,6259	41,0	83,0	30,6964
	90	121	236	24,3962	50,0	101,0	24,762

spoke about the repercussions that these new discoveries may have. The Wolfe line search approach was used in order for us to identify its worldwide convergence. Simulation-based research has shown that BN1, BN2, and BN3 have the ability to significantly reduce the number of iterations and function evaluations while maintaining the same degree of picture quality.

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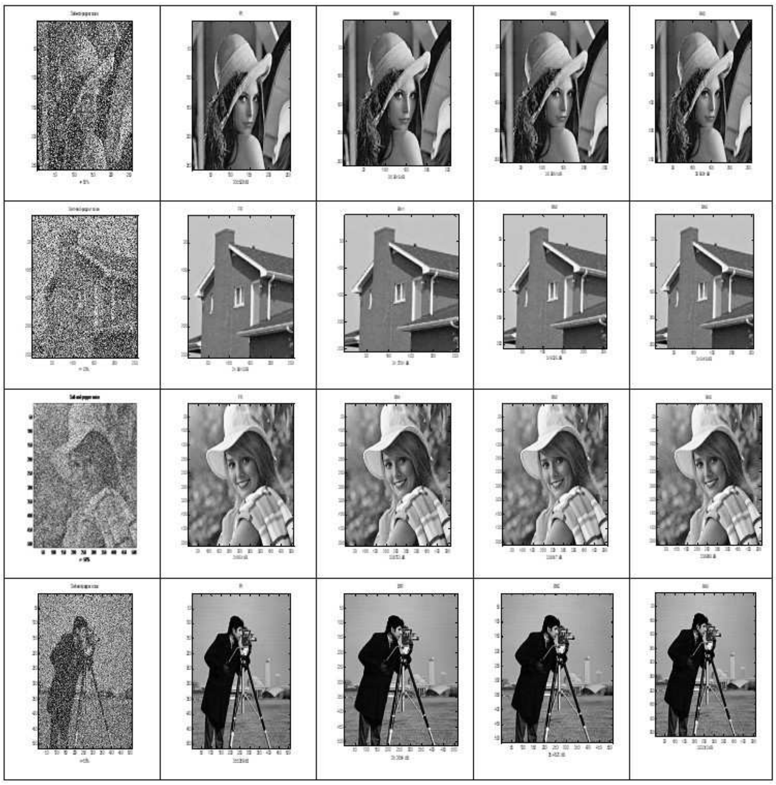

Fig. 1. Demonstrates the results of algorithms FR, BN1, BN2 and BN3 of 256 * 256 images

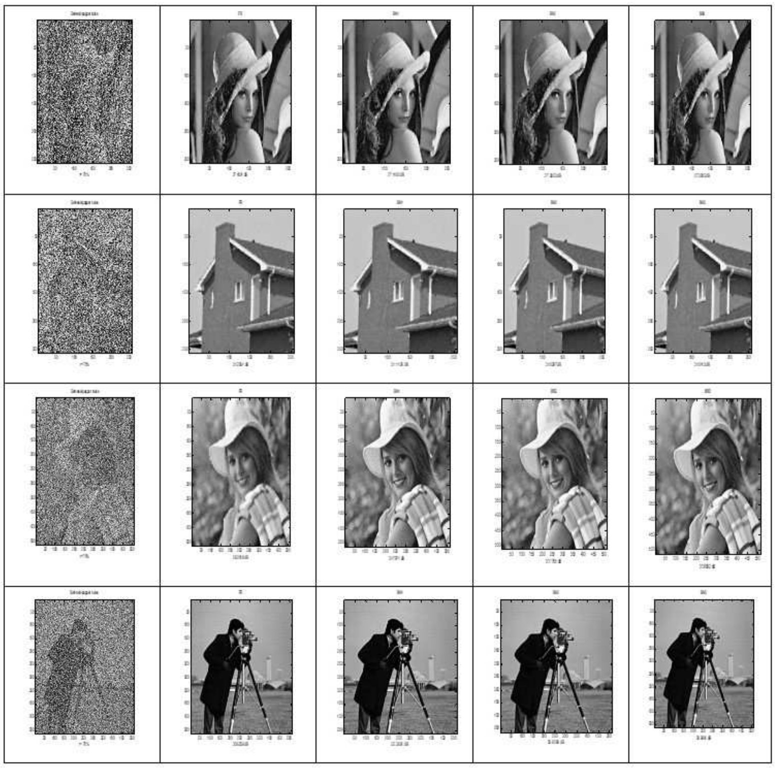

Fig. 2. Demonstrates the results of algorithms FR, BN1, BN2 and BN3 of 256 * 256 images

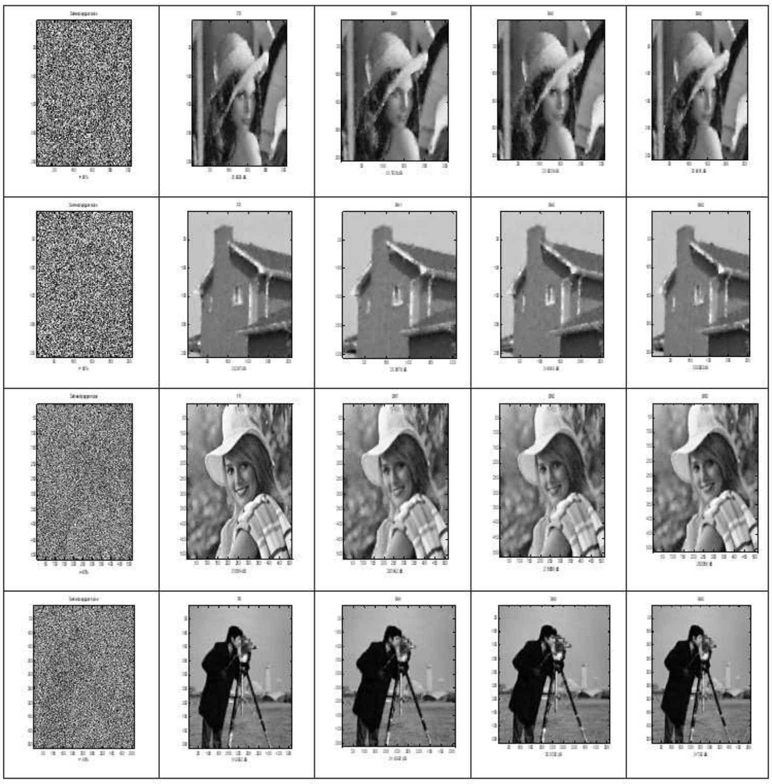

Fig. 3. Demonstrates the results of algorithms FR, BN1, BN2 and BN3 of 256 * 256 images

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ВЛИЯНИЕ НА НОВЫЕ ФОРМУЛЫ ГРАДИЕНТА ДЛЯ УДАЛЕНИЯ ИМПУЛЬСНЫХ ШУМОВ ИЗОБРАЖЕНИЙ

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В методах сопряженных градиентов формула сопряжения часто является основной точкой концентрации. Техника сопряженных градиентов используется для решения проблем, возникающих в процессе восстановления изображения. Используя квадратичную модель, для операции будет получено совершенно новое сопряжение коэффициентов. Алгоритмы демонстрируют как локальную, так и глобальную сходимость и спуск. Численное тестирование показало, что недавно разработанный метод намного превосходит тот, который существовал до него. Недавно созданная стратегия сопряженного градиента имеет более высокую производительность, чем метод сопряженного градиента FR, который является отраслевым стандартом.

Ключевые слова: влияние на градиент формулы; свойство конвергенции; импульсное шумоподавление изображений.

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