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INVESTIGATION OF THE TRANSIENT RESPONSES OF A BEAM ON AN ELASTIC POLYMERIC FOUNDATION

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The negative impact of vibrations on various devices and mechanisms can be significant, so it is important to take this factor into account when designing, operating and maintaining various equipment and engineering systems. Various methods and technologies can be used to protect against the negative effects of vibrations. Special damping materials are often used. This research paper is devoted to the analysis of the effectiveness of vibration reduction taking into account the physical parameters of elastic polymeric materials. To conduct the study, a mathematical model describing motion of the beam resting on an elastic polymeric foundation is constructed. The model is based on a system of nonlinear differential equations. An algorithm was developed and applied for the numerical solution of this system of equations. Numerical experiments were carried out for the study of the system reaction to different cases of accelerations. As a result, the deflection structure for materials with different physical characteristics were obtained. These results can serve as a starting point for a deeper study of materials and creation of more complex structures.

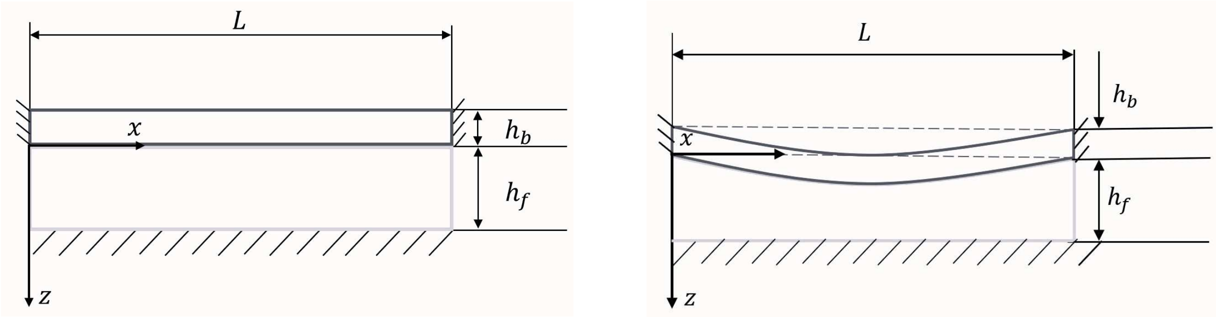
Keywords: transient responses; elastic polymeric foundation; nonlinear differential equation; Galerkin method.

Introduction

Vibrations can have a negative impact on various devices and mechanisms. Numerous studies in various fields consider the methods of reducing the impact of vibrations. Vibration reduction is an important issue in the structures and buildings, the transport sector, as well as in the manufacture of precision equipment [1–4]. The classic way to reduce vibrations is to use elastic polymeric layers. Elastomers are commonly used for damping due to their high elasticity and damping properties. The damping efficiency depends on many factors [5]. An important factor is the initial compression stress and the area of the elastic polymeric element. However, the choice of damping material is initially important. Rubber used to be the most common material that can reduce vibration. But at the moment, many new materials have appeared, such as elastomers. In practice, dielectric elastomers (DE) [6, 7], magnetorheological elastomers (MRE) [8] and thermoplastic elastomers (TPE) [9] are often used. However, researchers are now developing new materials for vibration isolation. This work is aimed at investigating the effectiveness of vibration isolation depending on the main characteristics of elastic polymeric materials – Young's modulus and material density.

1. Model Specification

Fig. 1 illustrates a structure that consists of an elastic beam and an elastic polymeric foundation. The elastic polymeric foundation is a damping layer. Fig. 1a provides a side view of the structure with dimensions: length L, width b, beam thickness h_b and elastic



a) b)

Fig. 1. Side view of the structure a) before the force is applied, b) after the force is applied

polymeric foundation thickness h_f, before it is subjected to external acceleration a_0. Fig. 1b presents a side view after the force or external acceleration is applied.

This allows us to present the governing equation for the transverse motion of the beam as a nonlinear integro-differential equation:

$$N_1(w_b, a_0) = E_b I \frac{\partial^4 W_b}{\partial x^4} - \frac{E_b A}{2L}\left[\int_0^L (\frac{\partial W_b}{\partial x})^2 dx\right] \frac{\partial^2 W_b}{\partial x}^2 + \\ +\rho_b A\left(\frac{\partial^2 W_b}{\partial t^2} + a_0\right) + c_b \frac{\partial W_b}{\partial t} + F_f(x,t) = 0, \tag{1}$$

where E_b is the effective Young's modulus of the beam, $W_b(x,t)$ is the deflection of the beam, defined to be positive in the direction to the right, $A = bh$ is the cross-section area, $I = \frac{1}{12}dh_b^3$ is the moment of inertia of the beam cross section, b is the beam width and h_b is the beam thickness, $a_0(t)$ is the external acceleration field, c_b is equivalent damping coefficients of the beam in transversal vibration of the structure, ρ_b is a density of the beam. F_f represents the force exerted by the foundation on the beam:

$$F_f(x,t) = -bE_f^{(0)}\left[\frac{\partial W_f}{\partial z} + \frac{1}{2}\frac{\partial W_f}{\partial z}^2\right]\Bigg|_{z=0}.$$

Boundary conditions for equation (1) are as follows:

$$W_b(0,t) = 0, \qquad \frac{\partial W_b}{\partial t}\Bigg|_{x=0} = 0, \qquad W_b(L,t) = 0, \qquad \frac{\partial W_b}{\partial t}\Bigg|_{x=L} = 0. \tag{2}$$

Equation

$$N_2(w_f) = \rho_f \frac{\partial^2 W_f}{\partial t^2} + c_f \frac{\partial W_f}{\partial t} - E_f\left(\frac{\partial^2 W_f}{\partial z^2} + \frac{\partial^2 W_f}{\partial z^2}\frac{\partial W_f}{\partial z}\right) = 0 \tag{3}$$

describes transverse motion of the damping layer. Here $W_f(x,z,t)$ and E_f are the deflection and effective Young's modulus of the elastic polymeric layer respectively, ρ_f and c_f represent the density and equivalent damping coefficients of the elastic polymeric layer accordingly.

Boundary conditions for equation (3) are as follows:

$$W_f(x,0,t) = W_b(x,t), \qquad W_f(x,h_f,t) = 0. \tag{4}$$

2. Numerical Solution

The Galerkin method was used to study the behavior of the transverse motion of the beam associated with the compressive motion of the polymer layer of the structure under the influence of an external force. Partial nonlinear equations of mixed form were discretized using this method, turning the system into a discrete number of ordinary differential equations depending on the considered number of spatial basis functions. Since the accompanying boundary condition for the compressive motion of the elastic polymeric layer is mobile, we consider the following transformation for the compressive motion of the polymeric layer:

$$W_f(x,z,t) = U(x,z,t) + \left(1 - \frac{z}{h_f}\right) W_b(x,t). \tag{5}$$

Introduce the following dimensionless quantities:

$$w_b = \frac{W}{h_f},\ u = \frac{U}{h_f},\ \xi = \frac{x}{L},\ \zeta = \frac{z}{h_b},\ \tau = \frac{t}{t^*}, \tag{6}$$

where t^* is a time scale, $t^* = \sqrt{\frac{\rho A L^4}{E_b I}}, \tau = \frac{t}{t^*}$ is the dimensionless time respectively.

Then the solutions of system (1), (3) can be expressed as follows in terms of the basis functions, satisfying the boundary conditions (2), (4):

$$w_b(\xi,\tau) = \sum_{n=1}^{N} a_n(\tau)\phi_n(\xi), \tag{7}$$

$$u(\xi,\zeta,\tau) = \sum_{n=1}^{N}\sum_{m=1}^{M} b_{nm}(\tau)\phi_n(\xi)\psi_m(\zeta), \quad n = 1,2,3,\ldots,N,\ m = 1,2,3,\ldots,M. \tag{8}$$

The error functions are constructed by substituting solutions (6), (7) and (8) into the system of equations (1) and (3). As a result, we obtain nonlinear ordinary differential equations that describe the motion of the structure:

$$\begin{gathered}\sum_{n=1}^{N} M^{(1)}_{nk}\ddot{a}_n + \sum_{n=1}^{N} C^{(1)}_{nk}\dot{a}_n + \sum_{n=1}^{N} K^{(1)}_{nk} a_n + \sum_{m=1}^{M}\sum_{n=1}^{N} E^{(1)}_{nmk} b_{nm} + \\ + \sum_{m=1}^{N}\sum_{n=1}^{N} Z^{(1)}_{nmk} a_n a_m + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{N} L^{(1)}_{nmpk} b_{nm} a_p + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{M}\sum_{s=1}^{N} F^{(1)}_{nmpsk} b_{nm} b_{pq} + \\ + \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{p=1}^{N} N^{(1)}_{nmpk} a_n a_m a_p + \sum_{k=1}^{N} D^{(1)}_1 a_0(\tau) = 0,\end{gathered} \tag{9}$$

$$\begin{gathered}\sum_{m=1}^{M}\sum_{n=1}^{N} M^{(2)}_{mnkq}\ddot{b}_{mn} + \sum_{n=1}^{N} M^{(3)}_{nkq}\ddot{a}_n + \sum_{m=1}^{M}\sum_{n=1}^{N} C^{(2)}_{mnkq}\dot{b}_{mn} + \sum_{m=1}^{M}\sum_{n=1}^{N} K^{(2)}_{mnkq} b_{mn} + \\ + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{M}\sum_{s=1}^{N} N^{(2)}_{mnpskq} b_{mn} b_{ps} + \sum_{n=1}^{N} E^{(2)}_{nkq}\dot{a}_n + \\ + \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{p=1}^{N} G^{(2)}_{mnpkq} b_{mn} a_p = 0,\end{gathered} \tag{10}$$

where the coefficients $M_{nk}^{(1)}, C_{nk}^{(1)}, K_{nk}^{(1)}$ and so on are constructed by scalarly multiplying the error equations in $L_2(G)$.

3. Numerical Experiments

In this section of the article, we interpret the findings of this study. We consider several different cases and explain the need to use elastic polymeric layers. The cases will be aimed at studying the relationship between deflection and material properties.

According to the model described previously, the elastic beam and the elastic polymeric layer have the properties described in Table 1 and Table 2. It is assumed that the elastic beam is made of steel, and the elastic polymeric layer is made of elastic material. It is worth noting that we do not use specific materials in the study. The study is based on changes in the Young's coefficient and the density of the material. These two parameters are among the main ones in the description of elastic polymeric foundation.

Table 1

Material and geometrical properties of elastic beam

Length L	$100\mu m$
Thickness h_b	$1\mu m$
Width b	$10\mu m$
Effective Young's modulus E_b	$169GPa$
Density ρ_b	$2330kg/m^3$
Equivalent damping coefficients c_b	$0,052 - 0,054$

Table 2

Geometrical properties of elastic polymeric layer

Length L	$100\mu m$
Thickness h_b	$5\mu m$
Width b	$10\mu m$

The basis functions are linearly independent and satisfy boundary conditions (2), (4) as well as the completeness condition. The following functions are selected as basis functions:

$$\phi_n(\xi) = 1 - \cos(2\pi n\xi), \qquad \psi_n(\zeta) = \sin(\pi n\zeta).$$

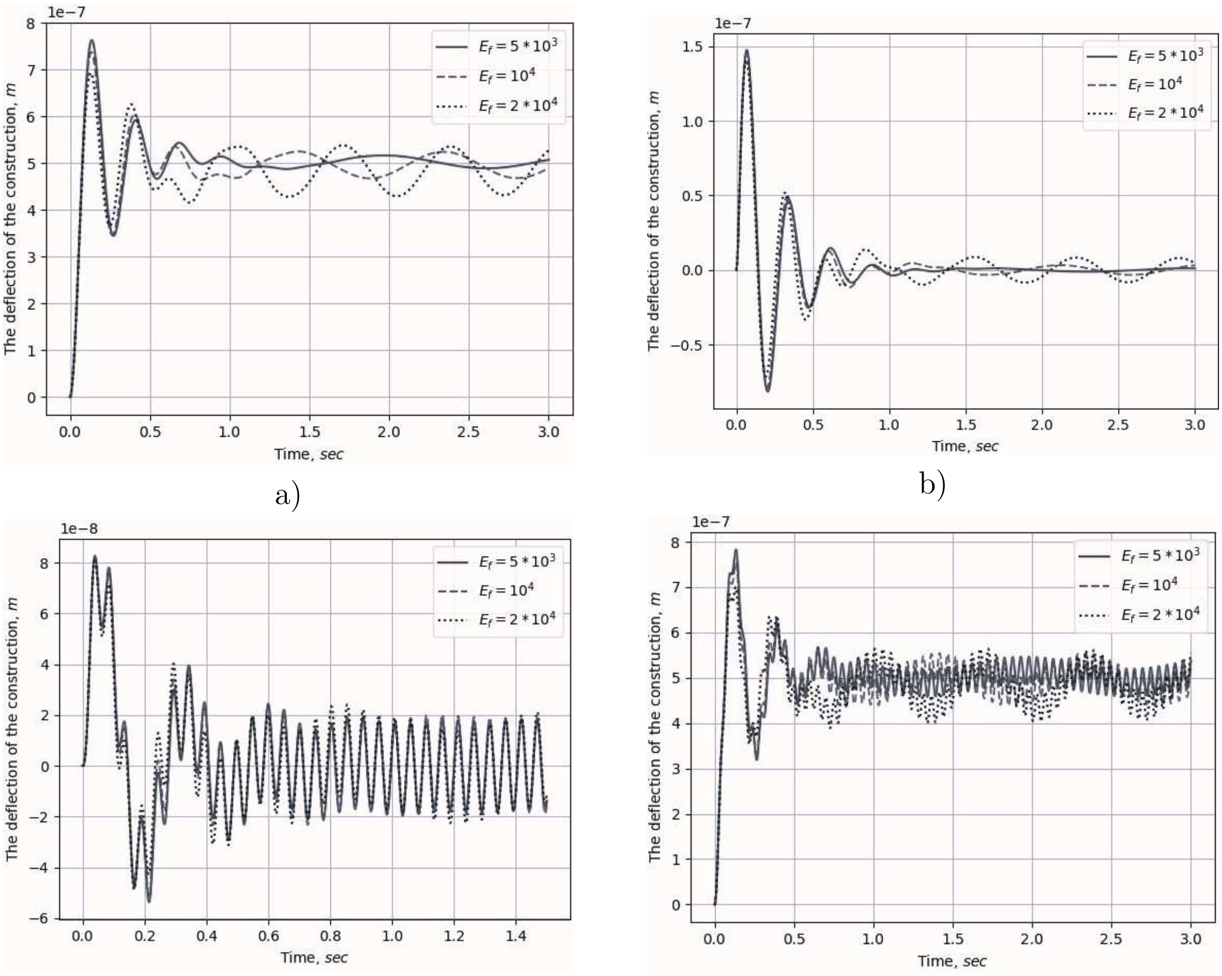
The applied force holds the structure in vertical position. The purpose of experiments is to study the deflection structure (w_b and w_f) when subjected to different type of accelerations a_0 over time t.

An algorithm for conducting computational experiments has been developed and implemented in Python. The program calculates the coefficients for equations (9), (10). The system of nonlinear differential equations is solved by the Runge–Kutta method of the 4-th order.

The following acceleration parameters were selected for the experiment:

$$a_0 = 60\frac{m}{sec^{-2}}, \quad \alpha = 100\frac{m}{sec^{-2}}, \quad \omega_n = 122sec^{-1}.$$

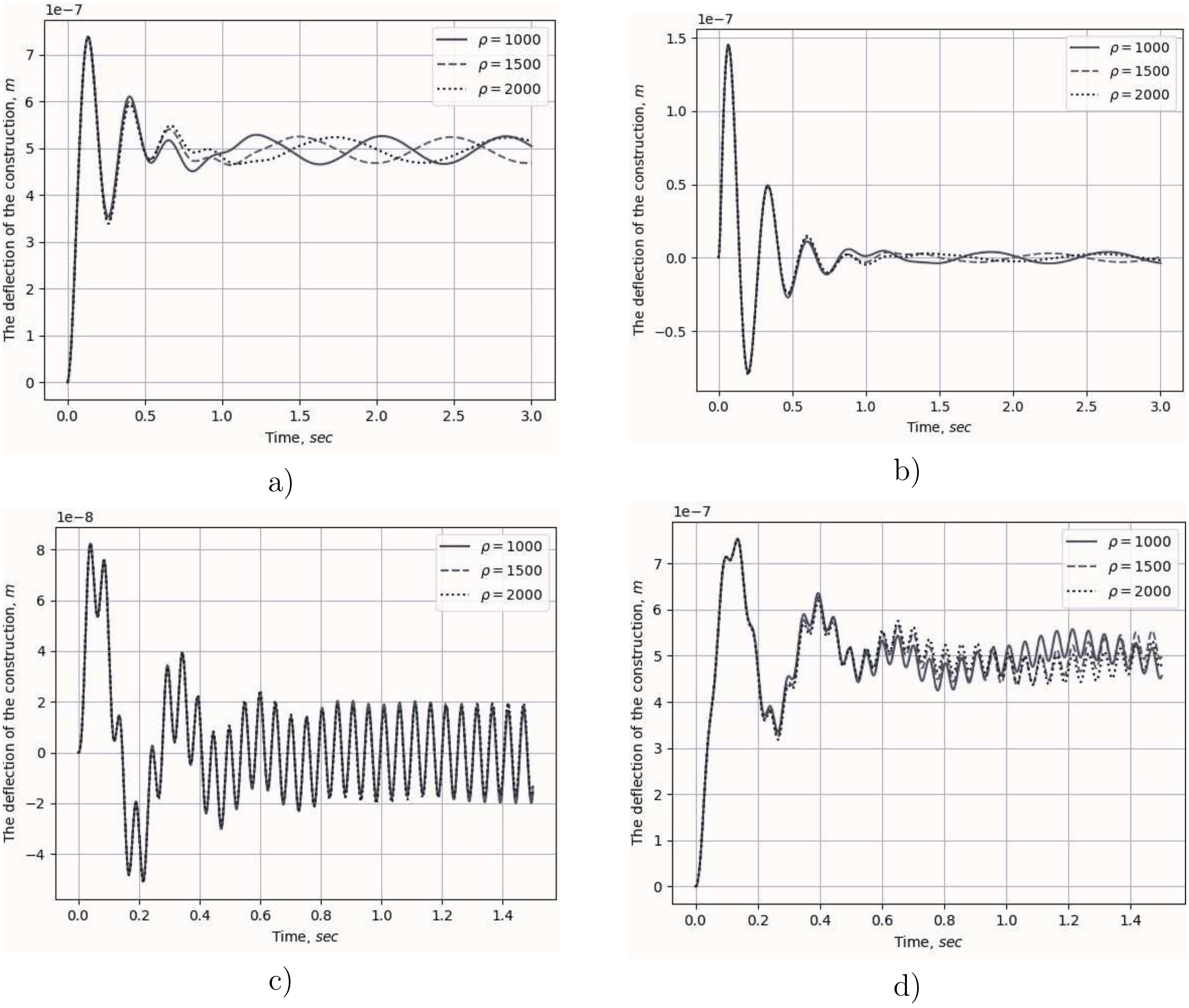
Fig. 2 illustrates the effect of acceleration $a_0(\tau)$ on the deflection of the elastic beam $w_b(\xi,\tau)$ and elastic polymeric layer $w_f(\xi,\zeta,\tau)$ when effective Young's modulus $E_b = 5kPa, 10kPa, 20kPa$. The deflection value is determined precisely at the point of contact between the beam and the layer, where: $\xi = 0,5, y = 0, \zeta = 0$. As a result of the acceleration the beam experiences a strong initial force, resulting in a large displacement. However, as time progresses, the amplitude of these oscillations gradually diminishes. This gradual decrease in amplitude can be attributed to various factors. One key aspect is the inherent damping properties of the elastic polymeric layer involved. Graphs in Fig. 2 suggest that when the coefficient changes, the structure has different oscillation amplitudes. The higher is the coefficient, the greater is the amplitude of vibrations of the structure.



a) b) c) d)

Fig. 2. The non-dimensional deflection of the structure under the influence of acceleration a_0 at the central point of connection of the elastic beam and the damping layer: a) $a_0(\tau) = a_0 H(\tau)$, where $H(\tau)$ is the Heaviside function; b) $a_0(\tau) = a_0\delta(\tau)$, where $\delta(\tau)$ is the Dirac delta function; c) $a_0(\tau) = a_0 \sin(\omega_n \tau)$; d) $a_0(\tau) = a_0 + \alpha \sin(\omega_n \tau)$

The effects of changes in the density of an elastic polymeric material on the amount of bending of the structure were also examined. Fig. 3 represents the deflection of the elastic beam w_b and elastic polymeric layer w_f under the influence of acceleration when the material density $\rho = 1000, 1500$ and $2000 kg/m^3$. As shown by Fig. 3, a change in the density of the material has a negligible effect on the magnitude of the oscillation amplitude. This study represents the first step towards a deeper understanding of the properties of elastic polymeric materials. Further work is required to fully study the various materials used in the structure. In the future, additional research will be conducted, focusing on different materials, in order to determine their potential to be used in the described structure. This allows to determine more accurately the optimal materials to achieve the required characteristics and maximum efficiency in the structure.



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Conclusion

Vibration is a common phenomenon that can have a negative impact on the operation of various systems and devices. It can cause wear and damage to components, as well as lead to loss of energy and suboptimal operation of the equipment. Therefore, it is crucial to develop methods and materials that will reduce vibration and improve system performance. One of ways for improvement is to use elastic polymeric materials. Elastomers have the ability to absorb energy and reduce vibration transmission. Such materials are widely used in various fields, including the transportation, energy and building industries. In this article, a mathematical model that describes motion of the beam resting on an elastic polymeric layer is constructed. For this aim, nonlinear differential equations were used. They take into account the physical characteristics of elastic polymeric materials and their vibroisolating ability. The algorithm developed for the numerical solution of the system allows to calculate the deflection of the elastic beam and elastic polymeric layer under the influence of acceleration and take into account the features of elastomeric materials. Numerical experiments exploring the reaction of the system to external acceleration were carried out. The obtained results made it possible to determine the effectiveness of vibration isolation when using elastic polymeric materials with different physical characteristics. The values of the deflections of the structure for each of the materials are obtained. This gives information on which material can be most effective in a particular situation. These results represent a starting point for deeper research into elastic polymeric materials and the creation of more complex structures. Their analysis can help to improve system performance, reduce energy losses, and improve equipment reliability. The results of the study can also be used in engineering and in the design of various devices where vibration is a problem.

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ИССЛЕДОВАНИЕ ПЕРЕХОДНЫХ ХАРАКТЕРИСТИК БАЛКИ НА ЭЛАСТИЧНОМ ПОЛИМЕРНОМ ОСНОВАНИИ

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Негативное воздействие вибраций на различные устройства и механизмы может быть значительным, поэтому важно учитывать этот фактор при проектировании, эксплуатации и техническом обслуживании различного оборудования и инженерных систем. Для защиты от негативного воздействия вибраций могут использоваться различные методы и технологии. Часто используются специальные демпфирующие материалы. Данная исследовательская работа посвящена анализу эффективности снижения вибрации с учетом физических параметров эластомерных материалов. Для проведения исследования построена математическая модель, описывающая движение балки, опирающейся на эластомерное основание. Модель основана на системе нелинейных дифференциальных уравнений. В ходе работы был разработан и применен алгоритм численного решения этой системы уравнений. Были проведены численные эксперименты для изучения реакции системы на различные случаи ускорений. В результате были получены величины прогиба для материалов с различными физическими характеристиками. Эти результаты могут послужить отправной точкой для более глубокого изучения материалов и создания более сложных конструкций.

Ключевые слова: переходные процессы; эластичное полимерное основание; нелинейное дифференциальное уравнение; метод Галеркина.

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