SOME INVERSE PROBLEMS
FOR CONVECTION-DIFFUSION EQUATIONS

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We examine the well-posedness questions for some inverse problems in the mathematical models of heat-and-mass transfer and convection-diffusion processes. The coefficients and right-hand side of the system are recovered under certain additional overdetermination conditions, which are the integrals of a solution with weights over some collection of domains. We prove an existence and uniqueness theorem, as well as stability estimates. The results are local in time. The main functional spaces used are Sobolev spaces. These results serve as the base for justifying of the convergence of numerical algorithms for inverse problems with pointwise overdetermination, which arise, in particular, in the heat-and-mass transfer problems on determining the source function or the parameters of a medium.

Keywords: parabolic system; convection-diffusion; heat-and-mass transfer; inverse problem; control problem; boundary value problem; well-posedness.

Introduction

We examine the question on recovering of the right-hand side and coefficients in a second order convection-diffusion system. Let G be a domain in $\mathbb{R}^n$ with boundary Γ of class C^2 and let $Q = G \times (0, T)$. This system is of the form

$$u_t + A(t, x, D)u = f_c, \ (t, x) \in Q, \tag{1}$$

where A is a second order elliptic operator with matrix coefficients of dimension $h \times h$. The equation (1) is supplemented with the initial and boundary conditions

$$u|_{t=0} = u_0, \ \ u|_S = g(t, x), \tag{2}$$

where $S = (0, T) \times \Gamma$. The right-hand side and the operator A in (1) are of the form

$$f_c = \sum_{i=1}^{r_0} b_i(t, x) q_i(t) + f, \tag{3}$$

$$A(t, x, D)u = A = \sum_{i=r_0+1}^{r} q_i(t) A_i(x, t) + A_{r+1}(x, t), \ A_i u = \sum_{|\alpha| \le 2} a_\alpha^i D^\alpha u. \tag{4}$$

The unknowns in (1), (2) are a solution u and the functions $q_i(t)$ $(i = 1, 2, \ldots, r)$ occurring into the right-hand side of (1) and the operator A. We consider the following overdetermination conditions:

$$\int_{G_i} u \varphi_i(x) dx = \psi_i(t), \ i = 1, 2, \ldots, s, \ r = sh, \tag{5}$$

where $G_i \subset G$ are some domains. Consider also the overdetermination conditions

$$u(x_i, t) = \psi_i(t), \; x_i \in G, \; i = 1, 2 \ldots, s, \quad r = sh. \tag{6}$$

The problem of this type arise when describing heat and mass transfer, convection-diffusion, and filtration processes (see [1–5]). Inverse problems on recovering of the coefficients of an equation (1), depending on t with the overtermination conditions (5), where $r = 1$ and $G_i = G$, are exposed in [6–12]. The linear inverse problems on recovering of the right-hand side are studied in [5, 13] respectively. Similarly, both types of problems with conditions (5) and (6) are examined in [4, 14] and [15, 16]. A large number of physical statements and numerical methods of solving of the above-mentioned inverse problems with condition (6) is exposed in [27] and [28]. The problem on recovering of the source function (3) with the overdetermination conditions of the form (6) can be found in [29, Ch. 3], where the main attention is payed to numerical methods. In this monograph the problem of determination of the source function $G(x, t)$, with given measurements (6) is examined. Here, the source function is replaced with its approximation of the form (3) which is calculated numerically. Note that most articles are devoted to the case of some model equations, where $n = 1$. We can note only articles [17, 18], where problems (1), (2), (6) in general statement are treated. We can also refer to monographs [2, 6, 14, 19, 20], where the reader can find statements of inverse problems for parabolic equations and systems and the corresponding existence and uniqueness theorems as well as some numerical methods for inverse problems solving.

In the present article under natural conditions on the data of the problem, we demonstrate that problem $(1) - (5)$ is uniquely solvable and establish stability estimates for solutions. On one hand, problem $(1) - (5)$ is of interest in its own right. On the other hand, a solution to problem $(1) - (4)$, (6) can be approximated by solutions to the problem $(1) - (5)$ for a suitable choice of the weights $\varphi_i = \varphi_i(x, \varepsilon)$ depending on a parameter $\varepsilon > 0$ (actually, we can construct an approximation of the Dirac δ-function). The convergence can be established under appropriate conditions on the data. The latter fact allows to construct numerical algorithms for solving of problem $(1) - (4)$, (6). We should note that most of the authors for numerical solution of the inverse problems use methods based on minimization of some functional (which is not convex) (see, for instance, [29] или [27]). Algorithms relying on an approximation of solutions to the problem $(1) - (4)$, (6) by solutions to the problem $(1) - (5)$ allow to construct simpler methods and this fact is confirmed by numerical experiments. In the next section, we present some auxiliary statements and conditions on the data. In section 3 we state and prove our main results (Theorems 4, 5).

1. Definitions and Auxiliary Statements

Let E be a Banach space. The symbol $L_p(G; E)$ (G is a domain in $\mathbb{R}^n$) stands for the space of strongly measurable functions $u(x)$ defined on G with values in E such that the norm $\| \|u(x)\|_E \|_{L_p(G)}$ is finite [21]. We also use the spaces $C^k(\overline{G})$ comprising functions having derivatives up to the order k inclusively in G continuous in G and admitting continuous extensions onto $\overline{G}$. The notations of the Sobolev spaces $W_p^s(G; E)$, $W_p^s(Q; E)$ and so on are conventional (see [21, 22]). If $E = \mathbb{C}$ or $E = \mathbb{C}^n$ then we use the notation $W_p^s(G)$ or $C^k(\overline{G})$ rather than $W_p^s(G; E)$ or $C^k(\overline{G}; E)$. Thus, the inclusion $u \in W_p^s(G)$ (or $u \in C^k(\overline{G})$) for a given vector-function $u = (u_1, u_2, \ldots, u_k)$ means that every component

of u_i belongs to $W_p^s(G)$ (or $C^k(\overline{G})$). In this case the norm of the vector-function is the sum of the norms of the coordinates. A similar convention is used for matrices as well. Given an interval $J = (0, T)$, assign $W_p^{s,r}(Q) = W_p^s(J; L_p(G)) \cap L_p(J; W_p^r(G))$ and, respectively, $W_p^{s,r}(S) = W_p^s(J; L_p(\Gamma)) \cap L_p(J; W_p^r(\Gamma))$. In what follows the symbol ∇u stands for the vector $(u_{x_1}, u_{x_2}, \ldots, u_{x_n})$, i. e. the gradient of u in the space variables. The condition $\Gamma \in C^\alpha$ ($\alpha \geq 1$) is understood conventionally (see [25]).

The smoothness and consistency conditions. Fix $p > n + 2$. We assume that

$$u_0(x) \in W_p^{2-2/p}(G), \quad g(x,t) \in W_p^{1-1/2p,2-1/p}(S), \quad g(x,0) = u_0(x)|_{\partial G}, \tag{7}$$

$$\psi_i(t) \in C^1([0,T]), \ \psi_i(0) = \int_{G_i} u_0(x)\varphi_i(x)dx, \ i = 1,2,\ldots,s, \tag{8}$$

$$a_\alpha^i(t,x) \in L_\infty(Q) \ (|\alpha| < 2), \quad a_\alpha^i \in C(\overline{Q}) \ (|\alpha| = 2). \tag{9}$$

We assume below that the boundaries of the domains $\{G_j\}$ ($j = 1, 2, \ldots, s$) those in (5) belongs to the class C^1. We employ the following conditions on the weights $\{\varphi_j(x)\}$:

$$supp \ \varphi_j \subset \overline{G_j}, \quad \varphi_j \in W_q^1(G_j) \ (\tfrac{1}{q} + \tfrac{1}{p} = 1), \ j = 1,2,\ldots,s, \tag{10}$$

$$supp \ \varphi_j \subset G_j, \quad \varphi_j \in L_1(G), \ j = 1,2,\ldots,s. \tag{11}$$

Let $G_0 = \cup_{j=1}^s G_j$, $Q_0 = G_0 \times (0,T)$. Under the condition (10) we require that

$$b_j, f \in C([0,T]; L_p(G_0)) \ (j = 1,2,\ldots,r_0), \ a_\alpha^i \in C([0,T], W_p^1(G_0)) \text{ for } |\alpha| = 2, \tag{12}$$

$$a_\alpha^i \in C([0,T], L_p(G_0)) \ (i = r_0 + 1, r_0 + 2, \ldots, r + 1) \text{ for } |\alpha| \leq 1. \tag{13}$$

If we replace (10) with (11) then we need the conditions

$$\nabla b_k, \nabla f \in L_p(Q_0), \quad \nabla u_0 \in W_p^{2-\frac{2}{p}}(G_0), \ \nabla a_\alpha^i(x,t) \in L_\infty(Q_0) \ (|\alpha| \leq 2), \tag{14}$$

$$a_\alpha^i \in C([0,T], L_\infty(G_0)) \ (|\alpha| \leq 1), \ b_k, \ f \in C([0,T]; L_\infty(G_0)) \ (k = 1,2,\ldots,r_0), \tag{15}$$

where $i = r_0 + 1, r_0 + 2, \ldots, r$. In the case of the problem (1)–(5) we define the matrix B of dimension $r \times r$ whose rows with the numbers from $(k-1)h + 1$ to kh, $(k = 1, 2, ..., s)$ are occupied by the $h \times r$-matrices with columns

$$\int_G b_1(x,0)\varphi_k dx, ..., \int_G b_{r_0}(x,0)\varphi_k dx, \ -\int_G A_{r_0+1}(x,0)u_0\varphi_k dx, \ldots, -\int_G A_r(x,0)u_0\varphi_k dx.$$

We require that

$$\det B \neq 0 \ \forall t \in [0,T]. \tag{16}$$

Assign $G_{\delta,i} = \{x \in G_i : \rho(x, \partial G_i) > \delta\}$, $Q_{\delta,i}^\gamma = G_{\delta,i} \times (0,\gamma)$, $G_\delta = \cup_{i=1}^s G_{\delta,i}$, and $Q_\delta = G_\delta \times (0,T)$, $Q_\delta^\gamma = G_\delta \times (0,\gamma)$ ($\delta \geq 0$), $Q^\gamma = G \times (0,\gamma)$. Let $A = \sum_{|\alpha| \leq 2} a_\alpha D^\alpha$, where a_α are $h \times h$-matrices. This expression can be also rewritten in the form $A = -\sum_{i,j=1}^n a_{ij}\partial_{x_i x_j} + \sum_{i=1}^n a_i \partial_{x_i} + a_0$. We say that A is elliptic whenever

$$\text{Re} \sum_{i,j=1}^n (a_{ij}(x,t)\xi^i, \xi^j) \geq \delta_0 \sum_{i=1}^n |\xi^i|^2, \quad \forall \xi^1, \xi^2, \ldots, \xi^n \in \mathbb{R}^h, \forall(x,t) \in Q. \tag{17}$$

Here δ_0 is some positive constant. The following theorems are valid.

Theorem 1. *Assume that G is a bounded domain with boundary of class C^2, the conditions (7) hold, the coefficients of the operator $A = \sum_{|\alpha| \leq 2} a_\alpha D^\alpha$ satisfy the condition*

$$a_\alpha(t, x) \in L_\infty(Q) \ (|\alpha| < 2), \ a_\alpha \in C(\overline{Q}) \ (|\alpha| = 2) \tag{18}$$

and the ellipticity condition (17) holds. If $f \in L_p(Q)$ then there exists a unique solution $u \in W_p^{1,2}(Q)$ to the problem

$$u_t + A(t, x, D_x)u = f, \quad u|_{t=0} = u_0(x), \ u|_S = g, \tag{19}$$

satisfying the estimate

$$\|u\|_{W_p^{1,2}(Q)} \leq c\Big[\|f\|_{L_p(Q)} + \|g\|_{W_p^{1-1/2p, 2-1/p}(S)} + \|u_0\|_{W_p^{2-2/p}(G)}\Big], \tag{20}$$

where c is a constant independent of f, u_0 and a solution u. If we additionally assume that

$$\nabla u_0 \in W_p^{2-\frac{2}{p}}(G_0), \ \nabla a_\alpha(x, t) \in W_\infty^1(Q_0), \ |\alpha| \leq 2, \tag{21}$$

and $\nabla f \in L_p(Q_0)$ then a solution u possesses the property $\nabla u \in W_p^{1,2}(Q_\delta)$ for all $\delta > 0$ and, for a fixed $\delta > 0$, we have the estimate

$$\|\nabla u\|_{W_p^{1,2}(Q_\delta)} + \|u\|_{W_p^{1,2}(Q)} \leq c\Big[\|f\|_{L_p(Q)} + \|\nabla f\|_{L_p(Q_0)} +$$
$$\|g\|_{W_p^{1-1/2p, 2-1/p}(S)} + \|u_0\|_{W_p^{2-2/p}(G)} + \|\nabla u_0\|_{W_p^{2-2/p}(G_0)}\Big]. \tag{22}$$

Proof. The first claim follows from Theorem 10.4 in [25]. The second claim is justified conventionally with the use of the finite difference method and Lemma 4.6 of Ch. 2 in [26].
□

Theorem 2. *Assume that G is a bounded domain with boundary of the class C^2, the coefficients of $A = \sum_{|\alpha| \leq 2} a_\alpha D^\alpha$ satisfy (18), the ellipticity condition (17) holds, and $f \in L_p(Q^\gamma)$ $(\gamma \in (0, T])$. Then there exists a unique solution $u \in W_p^{1,2}(Q^\gamma)$ to the problem*

$$u_t + A(t, x, D_x)u = g, \quad u|_{t=0} = 0, \quad u|_S = 0, \tag{23}$$

satisfying the estimate

$$\|u\|_{W_p^{1,2}(Q^\gamma)} \leq c\|g\|_{L_p(Q^\gamma)}, \tag{24}$$

where c is a constant independent of γ.

Proof. A solution sought agrees on $[0, \gamma]$ with a solution u_γ to the problem (19) with the right-hand side $g_\gamma = \begin{cases} g, & t \in [0, \gamma] \\ 0, & t \in (\gamma, T] \end{cases} \in L_p(Q)$ and the homogeneous initial and boundary conditions. Theorem 1 yields the estimate $\|u_\gamma\|_{W_p^{2,1}(Q)} \leq c\|g_\gamma\|_{L_p(Q)} \leq c\|g\|_{L_p(Q^\gamma)}$ which validates (24).
□

Theorem 3. *Assume that the conditions of Theorem 2 and condition (21) on a_α hold. Then a solution u to problem (23) for a fixed $\delta_1 > 0$ satisfies the estimate*

$$\|u\|_{W_p^{1,2}(Q^\gamma)} + \|\nabla u\|_{W_p^{1,2}(Q_{\delta_1}^\gamma)} \leq c(\|g\|_{L_p(Q^\gamma)} + \|\nabla g\|_{L_p(Q_0^\gamma)}), \tag{25}$$

where c is independent of γ.

Proof. The proof is in line with that of Theorem 2.

□

2. The Basic Results

Define the constants q_i^0 $(i = 1, 2, \ldots, r)$ as solutions to the system

$$\psi_{jt}(0) + \sum_{i=r_0+1}^{r} q_i^0 \int\limits_G A_i u_0 \varphi_j \, dx + \int\limits_G A_{r+1} u_0 \varphi_j \, dx = \sum_{i=1}^{r_0} q_i^0 \int\limits_G b_i(x, 0) \varphi_j \, dx + \int\limits_G f \varphi_j \, dx, \quad (26)$$

where $j = 1, 2, \ldots, s$. In the following theorem we assume that

$$\text{the operator } A^0 = \sum_{i=r_0+1}^{r} q_i^0 A_i + A_{r+1} \text{ is elliptic.} \quad (27)$$

Theorem 4. *Let the conditions* $(7) - (8), (9), (10), (12) - (13), (16), (27)$ *hold. Then there exists* $\gamma_0 > 0$ *such that, for* $t \in [0, \gamma_0]$, *there exists a unique solution* $(u, \vec{q})$ $(\vec{q} = (q_1, .., q_r))$ *to the problem* $(1) - (5)$ *such that* $u \in W_p^{1,2}(Q^{\gamma_0})$, $q_i(t) \in C([0, \gamma_0])$, $i = 1, 2, ..., r$. *A solution depends continuously on the data of the problem, i. e., for every two solutions* $(u^i, q_1^i, \ldots, q_r^i)$ $(i = 1, 2)$ *to the problem* $(1) - (5)$ *from the class*

$$u^i \in W_p^{1,2}(Q^{\gamma_0}), \ q_j^i \in C([0, \gamma_0]), \ (i = 1, 2, \ j = 1, 2, \ldots, r),$$

corresponding two different collections of the data f^i, ψ_j^i, u_0^i, g^i $(j = 1, 2, \ldots, s, \ i = 1, 2)$ *satisfying* $(7) - (8), (9) - (10), (12) - (13)$ *and such that the condition* (16) *holds and*

$$\|f^i\|_{L_p(Q)} + \sum_{j=1}^{s} \|\psi_j^i\|_{C^1([0,T])} + \|f^i\|_{C([0,T]; L_p(G_0))} +$$

$$+ \|u_0^i\|_{W_p^{2-2/p}(G)} + \|g^i\|_{W_p^{1-1/2p, 2-1/p}(S)} \leq R, \quad (i = 1, 2,)$$

there exists $\gamma_1 = \gamma_1(R)$ *such that the stability estimate*

$$\|u^1 - u^2\|_{W_p^{1,2}(Q^{\gamma_1})} + \sum_{j=1}^{r} \|q_j^1 - q_j^2\|_{C([0,\gamma_1])} \leq C(\|f^1 - f^2\|_{L_p(Q^{\gamma_1})} + \sum_{j=1}^{r} \|\psi_j^1 - \psi_j^2\|_{C^1([0,\gamma_1])} +$$

$$+ \|f^1 - f^2\|_{C([0,T]; L_p(G_0))} + \|u_0^1 - u_0^2\|_{W_p^{2-2/p}(G)} + \|g^1 - g^2\|_{W_p^{1-1/2p, 2-1/p}(S^{\gamma_1})}),$$

is valid, where C *is a constant independent of* R *and* γ_1 *and* $S^{\gamma_1} = \partial G \times (0, \gamma_1)$.

Proof. We construct a function Φ as a solution to the problem

$$\Phi_t + (\sum_{i=r_0+1}^{r} q_i^0 A_i + A_{r+1})\Phi = f + \sum_{i=1}^{r_0} q_i^0 b_i(x, t), \ \Phi|_{t=0} = u_0(x), \ \Phi|_S = g. \quad (28)$$

A solution to this problem exists and possesses the properties those of Theorem 1. Make the change $V = u - \Phi$ and $\vec{q} = \vec{q^1} + \vec{q^0}$ $(\vec{q^i} = (q_1^i, \ldots, q_r^i), \ i = 0, 1)$. In this case V is a solution to the problem

$$V_t + A^0 V + \sum_{i=r_0+1}^{r} q_i^1 A_i V = \sum_{i=1}^{r_0} b_i q_i^1 - \sum_{i=r_0+1}^{r} q_i^1 A_i \Phi, \ V|_{t=0} = 0, \ V|_S = 0. \quad (29)$$

Theorem 2 implies that for every $f \in L_p(Q^\tau)$ the problem

$$V_t + A^0 V = f, \ V|_{t=0} = 0, \ V|_S = 0 \quad (30)$$

is uniquely solvable and

$$\|V\|_{W_p^{1,2}(Q^\tau)} \le c\|g\|_{L_p(Q^\tau)}, \tag{31}$$

where the constant c is independent of $\tau \in [0, T]$. Examine an auxiliary problem

$$V_t + A^0 V + \sum_{i=r_0+1}^{r} q_i^1 A_i V = g(x, t), \quad V|_{t=0} = 0, \ V|_S = 0. \tag{32}$$

Let $\vec{q^1} \in B_{\mu,\tau} = \{\vec{q^1} \in C([0,\tau]) : \ \|\vec{q^1}\|_{C([0,\tau])} \le \mu\}$ $(\tau \le T)$. By Theorem 2, we have

$$V + (\partial_t + A^0)^{-1} \sum_{i=r_0+1}^{r} q_i^1 A_i V = (\partial_t + A^0)^{-1} g(x, t). \tag{33}$$

The estimate (31) yields

$$\|(\partial_t + A^0)^{-1} \sum_{i=r_0+1}^{r} q_i^1 A_i V\|_{W_p^{1,2}(Q^\tau)} \le c\| \sum_{i=r_0+1}^{r} q_i^1 A_i V\|_{L_p(Q^\tau)} \le$$
$$\|\vec{q^1}\|_{C([0,\tau])} cc_1 \|V\|_{W_p^{1,2}(Q^\tau)} \le \mu cc_1 \|V\|_{W_p^{1,2}(Q^\tau)}. \tag{34}$$

In what follows, we assume that $\mu \le \mu_0 = \frac{1}{2cc_1}$. In this case we infer

$$\|(\partial_t + A^0)^{-1}(\sum_{i=r_0+1}^{r} q_i^1 A_i V)\|_{W_p^{1,2}(Q^\tau)} \le \frac{1}{2}\|V\|_{W_p^{1,2}(Q^\tau)}$$

and thereby (33) is uniquely solvable and a solution $V \in W_p^{1,2}(Q)$ satisfies the estimate

$$\|V\|_{W_p^{1,2}(Q^\tau)} \le 2\|(\partial_t + A^0)^{-1} g\|_{W_p^{1,2}(Q^\tau)} \le 2c\|g\|_{L_p(Q^\tau)}, \ \forall \vec{q^1} \in B_{\mu,\tau}. \tag{35}$$

Integrating equation (29) with the weight φ_i over G, we obtain

$$\int_G V_t \varphi_i \, dx = \frac{\partial}{\partial t} \int_G V \varphi_i \, dx = \tilde{\psi}_{it}(t), \quad \tilde{\psi}_i(t) = \psi_i - \int_G \Phi \varphi_i \, dx,$$

$$\tilde{\psi}_{it} + \int_G A^0 V \varphi_i dx + \sum_{j=r_0+1}^{r} q_j^1 \int_G A_j V \varphi_i dx = \sum_{j=1}^{r_0} q_j^1 \int_G b_j \varphi_i dx - \sum_{j=r_0+1}^{r} q_j^1 \int_G A_j \Phi \varphi_i dx, \tag{36}$$

where $i = 1, 2, \ldots, s$. Examine the matrix $B(t)$ of dimension $r \times r$ whose rows with the numbers from $(k-1)h + 1$ to kh, $(k = 1, 2, \ldots, s)$ are occupied by the $h \times r$-matrices with the columns

$$\int_G b_1 \varphi_k dx, \ldots, \int_G b_{r_0} \varphi_k dx, -\int_G A_{r_0+1} \Phi \varphi_k dx, \ldots, -\int_G A_r \Phi \varphi_k dx.$$

Note that $B(0) = B$ with the matrix B that was defined before (16). Demonstrate that the entries of $B(t)$ are continuous in t. The continuity of the entries from the first r_0 columns results from (9). Consider the last $r - r_0$ columns. Since $\Phi \in W_p^{1,2}(Q)$, the embedding theorems yield

$$\Phi \in C([0, T]; W_p^\alpha(G)), \quad \alpha \le 2(1 - 1/p). \tag{37}$$

The latter results, for example, from Theorem 1.8.3 in [21], where the Banach spaces A_0, A_1 are replaced with $W_p^2(G)$ and $L_p(G)$. In view of our conditions on p, $2(1 - 1/p) > n/p + 1$ and, in particular, $W_p^{2(1-1/p)}(G) \subset C^\beta(\overline{G})$ with $\beta \leq 2 - 2/p - n/p$ [21]. Thus,

$$W_p^{1,2}(Q) \subset C([0,T]; W_p^{2(1-\frac{1}{p})}(G)) \subset C([0,T]; C^\beta(\overline{G})) \quad \beta \leq 2 - \frac{2}{p} - \frac{n}{p}. \qquad (38)$$

In particular, changing Φ on a set of a zero measure if necessary, we can assume that the functions $D_x^\alpha \Phi$ for $|\alpha| \leq 1$ are continuous. We have $A_j \Phi = A_{j0}\Phi + A_{j1}\Phi$ ($j > r_0$, where $A_{j0}\Phi = \sum_{|\alpha|=2} a_\alpha^j D^\alpha \Phi$ and $A_{j1}\Phi = \sum_{|\alpha|<2} a_\alpha^j D^\alpha \Phi$. In view of (13) and the above-pointed properties of Φ, for $|\alpha| \leq 1$ the functions $\int_G a_\alpha^j D^\alpha \Phi \varphi_i \, dx$ are continuous in t. Consider the case of $|\alpha| = 2$. We conclude that

$$\int_G a_\alpha^j(x,t) D^\alpha \Phi \varphi_i dx dt = \int_{\Gamma_i} a_\alpha(x,t) D^{\alpha'} \Phi \varphi_i n_k d\Gamma - \int_{G_i} (a_\alpha \varphi_i)_{x_k} D^{\alpha'} \Phi \, dx,$$

where $D^\alpha \Phi = \frac{\partial}{\partial x_k} D^{\alpha'} \Phi$, n_k is a coordinate of the unit outward normal to $\Gamma_i = \partial G_i$. By the same reasons, the obtained integrals are continuous in t.

In view of continuity of the entries of $B(t)$, there exists $\tau_0 > 0$ and a constant $\tilde{\delta} > 0$ such that $|det B(t)| > \tilde{\delta}$, $\forall t \in [0, \tau_0]$. The equalities (36) can be rewritten as

$$B(t)\vec{q^1} = \begin{bmatrix} \tilde{\psi}_{1t} + (A^0 V, \varphi_1) + \sum_{i=r_0+1}^r q_i^1 (A_i V, \varphi_1) \\ \vdots \\ \tilde{\psi}_{st} + (A^0 V, \varphi_s) + \sum_{i=r_0+1}^r q_i^1 (A_i V, \varphi_s) \end{bmatrix}, \; (u,v) = \int_G u\bar{v} dG.$$

Thus,

$$\vec{q^1} = g_0 + R(\vec{q^1}), \qquad (39)$$

where $g_0 = B(t)^{-1} \begin{bmatrix} \tilde{\psi}_{1t} \\ \vdots \\ \tilde{\psi}_{st} \end{bmatrix}$, $R(\vec{q^1}) = B(t)^{-1} \begin{bmatrix} (A^0 V, \varphi_1) + \sum_{i=r_0+1}^r q_i^1 (A_i V, \varphi_1) \\ \vdots \\ (A^0 V, \varphi_s) + \sum_{i=r_0+1}^r q_i^1 (A_i V, \varphi_s) \end{bmatrix}$ and V is

a solution to the problem (29), which exists whenever $\vec{q^1} \in B_{\mu,\tau}$, with $\mu \leq \mu_0$ и $\tau \leq T$. Consider the vector

$$g_0 = B(t)^{-1} \begin{bmatrix} \tilde{\psi}_{1t} \\ \vdots \\ \tilde{\psi}_{st} \end{bmatrix}.$$

Let $\tau \leq \tau_0$. We have $\|g_0\|_{C([0,\tau])} \leq c \max_{1 \leq i \leq s} \|\tilde{\psi}_{it}\|_{C([0,\tau])}$. In view of (26), $\tilde{\psi}_{it}(0) = 0$. By continuity of $\tilde{\psi}_{it}$, $\exists \tau_1 \leq \tau_0$: $\|g_0\|_{C([0,\tau])} \leq \frac{\mu_0}{2}$, $\forall \tau_0 \leq \tau_1$. Show that there exists $\tau_2 \leq \tau_1$ such that the equation (39) is uniquely solvable in the ball $B_{\mu_0,\tau_2} = \{\vec{q^1} : \|\vec{q^1}\|_{C([0,\tau_2])} \leq \mu_0\}$. Derive estimates for the operator $R(\vec{q^1})$. Let $\vec{q^1}$, $\vec{q^2} \in B_{\mu_0,\tau}$, ($\tau \leq \tau_1$), and V^1, V^2 are

solutions to problem (29) (with $\vec{q^1}$ and $\vec{q^2}$ on the right-hand side) satisfying the initial and boundary conditions $V^j|_{t=0} = 0$, $V^j|_S = 0$ $(j = 1, 2)$. We have

$$V_t^j + A^0 V^j + \sum_{j=r_0+1}^{r} q_i^j A_i V^j = \sum_{i=1}^{r_0} q_i^j b_i - \sum_{i=r_0+1}^{r} q_i^j A_i \Phi, \ V^j|_{t=0} = 0, \ V^j|_S = 0, \qquad (40)$$

where $i = 1, 2, \ldots, m$, $j = 1, 2$. Estimate $\|R(\vec{q^1}) - R(\vec{q^2})\|_{C([0,\tau])}$. We obtain

$$\|R(\vec{q^1}) - R(\vec{q^2})\|_{C([0,\tau])} \leq c \max_{1 \leq j \leq s} \|(A^0(V^1 - V^2), \varphi_j) + \qquad (41)$$
$$\sum_{i=r_0+1}^{r} (q_i^1 (A_i V^1, \varphi_j) - q_i^2 (A_i V^2, \varphi_j))\|_{C([0,\tau])}.$$

Subtracting the equations (40) for $j = 1, 2$, we arrive at the equality

$$(V^1 - V^2)_t + A^0(V^1 - V^2) + \sum_{i=r_0+1}^{r} q_i^2 (A_i V^1 - A_i V^2) = \qquad (42)$$
$$\sum_{i=1}^{r_0} (q_i^1 - q_i^2) b_i - \sum_{i=r_0+1}^{r} (q_i^1 - q_i^2) A_i V^1 - \sum_{i=1}^{r_0} (q_i^1 - q_i^2) A_i \Phi.$$

Note that in view of (35) the functions V^j meet the estimate

$$\|V^j\|_{W_p^{1,2}(Q^\tau)} \leq 2C \| \sum_{i=1}^{r_0} q_i^j b_i - \sum_{i=r_0+1}^{r} q_i^j A_i \Phi \|_{L_p(Q^\tau)} \leq C_1 \|\vec{q^j}\|_{C([0,\tau])} \leq C_1 \mu_0, \qquad (43)$$

where C_1 is independent of x and τ. Estimate (35) and equalities (42) yield

$$\|V^1 - V^2\|_{W_p^{1,2}(Q^\tau)} \leq C_2 \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])} + \max_i \|A_i V^1\|_{L_p(Q^\tau)} C_3 \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])} \leq \qquad (44)$$
$$\|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])} (C_2 + C_3 \|V^1\|_{W_p^{1,2}(Q^\tau)}) \leq \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])} (C_2 + C_3 C_1 \mu_0).$$

Estimate the right-hand side of (41) using the inclusion $\varphi_j \in W_q^1(G_j)$. Consider $|(A^0 v, \varphi_j)|$ for $v = V^1 - V^2$. We have $A^0 v = A_0 v + A_1 v$, where $A_0 v = \sum_{|\alpha|=2} a_\alpha D^\alpha v$ and $A_1 v = \sum_{|\alpha|<2} a_\alpha D^\alpha v$. For $|\alpha| \leq 1$, we infer $|\int_G a_\alpha D^\alpha v \varphi_i dx| \leq c \|D^\alpha v\|_{L_\infty(G_0)} \leq c \|v\|_{W_\infty^1(G_0)}$. Thus,

$$| \int_G A_1 v \varphi_i dx | \leq c_1 \|v\|_{W_\infty^1(G)}. \qquad (45)$$

Consider the expression $\int_G A_0 v \varphi_i dx$. It contains the summands

$$\int_G a_\alpha(x,t) D^\alpha v \varphi_i dx dt = \int_{\Gamma_i} a_\alpha(x,t) D^{\alpha'} v \varphi_i n_k d\Gamma - \int_{G_i} (a_\alpha \varphi_i)_{x_k} D^{\alpha'} v dx,$$

where $D^\alpha v = \frac{\partial}{\partial x_k} D^{\alpha'} v$, n_k is the k-th coordinate of the unit normal to $\Gamma_i = \partial G_i$. The second integral is estimated as

$$\int_{G_i} |a_\alpha| |D^{\alpha'} v| \cdot |\varphi_{i x_k}| dx + \int_{G_0} |a_{\alpha x_k}| \cdot |D^{\alpha'} v| \cdot |\varphi_i| dx \leq$$

$$\leq M\|D^{\alpha'}v\|_{L_\infty(G_0)}\Big(\int_{G_i}|\varphi_{ix_k}|^q dx\Big)^{\frac{1}{q}} + M_1\|D^{\alpha'}v\|_{L_\infty(G_0)}\Big(\int_{G_i}\varphi_i^q dx\Big)^{\frac{1}{q}}.$$

Therefore, the second integral is estimates by $c_1\|v\|_{W_\infty^1(G)}$. Estimate the first integral with the use of the trace theorems (see, for instance, [21, 25, 26]). In view of (38) we can assume that $v \in C([0,T]; C^1(\overline{G}))$, and thus $a_\alpha \in C(\overline{Q_0})$ and

$$\Big|\int_{\Gamma_i} a_\alpha D^{\alpha'}v\varphi_i n_k d\Gamma\Big| \leq \|v\|_{W_\infty^1(G)}\|a_\alpha\|_{L_\infty(G_0)}\|\varphi_i\|_{L_q(\Gamma_i)} \leq c_2\|v\|_{W_\infty^1(G)}.$$

We involved the fact that $\varphi_i \in W_q^{1-1/q}(\Gamma_i)$ due to embedding theorems. These two inequalities ensure that there exists c_3 such that

$$\Big|\int_G A_0 v\varphi_i dx\Big| \leq c_3\|v\|_{W_\infty^1(G)}. \tag{46}$$

By the embedding theorems and the interpolation inequalities (see (38), [21]), we infer

$$\|v\|_{W_\infty^1(G)} \leq c_4\|v\|_{W_p^{1+\frac{n}{p}}(G)} \leq c_5\|v\|_{W_p^{2-\frac{2}{p}}(G)}^\theta\|v\|_{L_p(G)}^{1-\theta}, \quad \theta(2-\tfrac{2}{p}) = 1+\tfrac{n}{p}. \tag{47}$$

In this case relations (45), (46), (47) imply that

$$\max_{1\leq j\leq s}\|(A^0(V^1-V^2),\varphi_j)\|_{C([0,\tau])} \leq C_6\|v\|_{C([0,\tau];W_p^{2-\frac{2}{p}}(G))}^\theta\|v\|_{C([0,\tau];L_p(G))}^{1-\theta}. \tag{48}$$

As it was noted, $v \in C([0,\tau]; W_p^{2-2/p}(G))$ after a possible modification on a set of zero measure. Next, $v = \int_0^t v_\tau(x,\tau)d\tau$ and thereby

$$\|v\|_{C([0,\tau];L_p(G))} \leq \tau^{\frac{1}{q}}\|v_t\|_{L_p([0,\tau];L_p(G))}, \quad (1/p + 1/q = 1). \tag{49}$$

Therefore, $\|(Av,\varphi_j)\|_{C([0,\tau])} \leq C\tau^{\frac{1-\theta}{q}}\|v\|_{W_p^{1,2}(Q^\tau)}$. In this case we have

$$\|(A^0(V^1-V^2),\varphi_j)\|_{C([0,\tau])} \leq c_7\tau^{\frac{1-\theta}{q}}\|V^1-V^2\|_{W_p^{1,2}(Q^\tau)}, \tag{50}$$

where C_1 is independent of τ. Using (44), we obtain

$$\|(A^0(V^1-V^2),\varphi_j)\|_{C([0,\tau])} \leq c_8\tau^{\frac{1-\theta}{q}}\|\vec{q^1}-\vec{q^2}\|_{C([0,\tau])}, \tag{51}$$

Consider the second summand in (41). We have

$$\sum_{i=r_0+1}^r (q_i^1(A_iV^1,\varphi_j) - q_i^2(A_iV^2,\varphi_j)) =$$
$$\sum_{i=r_0+1}^r (q_i^1 - q_i^2)(A_iV^1,\varphi_j) + \sum_{i=r_0+1}^r q_i^2(A_iV^1 - A_iV^2,\varphi_j).$$

The summands (A_iV^1,φ_j), $(A_i(V^1-V^2),\varphi_j)$ are estimated as the expression $(A^0(V^1-V^2),\varphi_j)$ (see (50)). We have

$$\|\sum_{i=r_0+1}^r (q_i^1 - q_i^2)(A_iV^1,\varphi_j)\|_{C[0,\tau]} \leq \|\vec{q^1}-\vec{q^2}\|_{C[0,\tau]}\tau^{\frac{1-\theta}{q}}c_9\|V^1\|_{W_p^{1,2}(Q^\tau)} \leq$$
$$\tau^{\frac{1-\theta}{q}}C(\mu_0)\|\vec{q^1}-\vec{q^2}\|_{C([0,\tau])}. \tag{52}$$

Similarly, in view of (44), we infer

$$\| \sum_{i=r_0+1}^{r} q_i^2 (A_i(V^1 - V^2), \varphi_j) \|_{C[0,\tau]} \le c_{10} \tau^{\frac{1-\theta}{q}} \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])}. \tag{53}$$

Relations (51) – (53) imply that

$$\|R(\vec{q^1}) - R(\vec{q^2})\|_{C[0,\tau]} \le c_{11} \tau^{\frac{1-\theta}{q}} \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau])}, \ \forall q^i \in B_{\mu_0,\tau}.$$

Find $\tau_2 \le \tau_1$ such that $c_{11} \tau_2^{\frac{1-\theta}{q}} \le \frac{1}{2}$. In this case

$$\|R(\vec{q^1}) - R(\vec{q^2})\|_{C([0,\tau_2])} \le \frac{1}{2} \|\vec{q^1} - \vec{q^2}\|_{C([0,\tau_2])}. \tag{54}$$

Since $R(\vec{q^2}) = 0$ for $\vec{q^2} = 0$, we have

$$\|R(\vec{q^1})\|_{C([0,\tau_2])} \le \frac{1}{2} \|\vec{q^1}\|_{C([0,\tau_2])}. \tag{55}$$

In this case the operator $R_0(\vec{q^1}) = g_0 + R(\vec{q^1})$ is contractive and takes the ball B_{μ_0,τ_2} into itself. Indeed,

$$\|R_0(q)\|_{C([0,\tau_2])} \le \|g_0\|_{C([0,\tau_2])} + \|R(\vec{q^1})\|_{C([0,\tau_2])} \le \frac{\mu_0}{2} + \frac{\mu_0}{2} = \mu_0.$$

The fixed point theorem implies the existence of a solution $\vec{q^1}$ to equation (39) in this ball. By construction, V is a solution to problem (29). Assign $\gamma_0 = \tau_2$. Demonstrate that V satisfies the conditions $\int\limits_G V \varphi_j dx = \tilde{\psi}_i(t)$. Integrate equation (29) over G with the weight φ_i. We have

$$\frac{\partial}{\partial t} \int_G \varphi_i V dx + \int_G A^0 V \varphi_i dx + \sum_{j=r_0+1}^{r} q_j^1 \int_G A_j V \varphi_i dx = \sum_{j=1}^{r_0} q_j^1 \int_G b_j \varphi_i dx - \sum_{j=r_0+1}^{r} q_j^1 \int_G A_j \Phi \varphi_i dx.$$

The function q_j satisfies (36); subtracting the i-th equation from the previous equality, we obtain that $\left(\int\limits_G \varphi_i v dx - \tilde{\psi}_i \right)_t = 0$ or $\int_G \varphi_i v dx - \tilde{\psi}_i = (\int\limits_G \varphi_i v dx - \tilde{\psi}_i)|_{t=0} = 0$ in view of the consistency conditions. Thus, v is a solution to our problem. The stability estimate from the statement of the theorem was actually obtained in the proof and we omit the arguments.

$\square$

Theorem 5. *Let conditions* (7) − (9), (11), (14) − (16), (27) *hold. Then there exists $\gamma_0 > 0$ such that on the segment $[0,\gamma_0]$ there exists a unique solution $(u, q_1, .., q_r)$ to problem* (1) − (5) *such that*

$$u \in W_p^{1,2}(Q^{\gamma_0}), \ q_i(t) \in C([0,\gamma_0]), \ i = 1, 2, ..., r.$$

A solution continuously depends on the data of the problem, i. e., for a fixed $\delta_1 > 0$ and every two solutions $(u^i, q_1^i, \ldots, q_r^i)$ $(i = 1, 2)$ to the problem (1)–(5) *of the class*

$$u^i \in W_p^{1,2}(Q^{\gamma_0}), \ q_j^i \in C([0,\gamma_0]), \ (i = 1, 2, \ j = 1, 2, \ldots, r),$$

corresponding two different collections of the data f^i, ψ^i_j, u^i_0, g^i ($j = 1, 2, \ldots, s$, $i = 1, 2$) such that conditions (7) – (9), (11), (14) – (16), (27), (16) *hold and*

$$\|f^i\|_{L_p(Q)} + \sum_{j=1}^{s} \|\psi^i_j\|_{C^1([0,T])} + \|f^i\|_{C([0,T];L_\infty(G_0))} + \|\nabla f^i\|_{L_p(Q_0)}$$

$$\|u^i_0\|_{W_p^{2-2/p}(G)} + \|\nabla u^i_0\|_{W_p^{2-2/p}(G)} + \|g^i\|_{W_p^{1-1/2p,2-1/p}(S)} \le R, \quad i = 1, 2,$$

there exists a constant $\gamma_1 = \gamma_1(R)$ such that the stability estimate

$$\|u^1 - u^2\|_{W_p^{1,2}(Q^{\gamma_1})} + \|\nabla(u^1 - u^2)\|_{W_p^{1,2}(Q^{\gamma_1}_{\delta_1})} + \sum_{j=1}^{r} \|q^1_j - q^2_j\|_{C([0,\gamma_1])} \le$$

$$C(\|f^1 - f^2\|_{L_p(Q^{\gamma_1})} + \sum_{j=1}^{r} \|\psi^1_j - \psi^2_j\|_{C^1([0,\gamma_1])} + \|f^1 - f^2\|_{C([0,T];L_\infty(G_0))} + \tag{56}$$

$$\|\nabla(f^1 - f^2)\|_{L_p(Q_0)} + \|u^1_0 - u^2_0\|_{W_p^{2-2/p}(G)} +$$

$$\|\nabla(u^1_0 - u^2_0)\|_{W_p^{2-2/p}(G_0)} + \|g^1 - g^2\|_{W_p^{1-1/2p,2-1/p}(S^{\gamma_1})},$$

holds, where $S^{\gamma_1} = \partial G \times (0, \gamma_1)$ and the constant C is independent of R and γ_1.

Proof. The proof relies on the same scheme as that of the proof of the previous theorem. As before, the claim is reduced to the solvability of equation (39).

$\square$

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НЕКОТОРЫЕ ОБРАТНЫЕ ЗАДАЧИ ДЛЯ СИСТЕМ УРАВНЕНИЙ КОНВЕКЦИИ-ДИФФУЗИИ

С.Г. Пятков, Е.И. Сафонов

В настоящей статье мы рассмотрим вопросы корректности некоторых обратных задач для математических моделей процессов тепломассопереноса и конвекции-диффузии. Коэффициенты и правая часть системы восстанавливаются при выполнении некоторых дополнительных условий переопределения. Эти условия есть значения интегралов решения с весами по некоторой совокупностью областей. Доказаны теоремы существования и единственности и установлены оценки устойчивости. Полученные результаты являются локальными по времени. В качестве основных функциональных пространств используются пространства Соболева. Результаты служат основой при обосновании сходимости численных алгоритмов решения обратных задач с точечными условиями переопределения, которые возникают, в частности, в задачах тепломассопереноса об определении функции источников и параметров среды.

Ключевые слова: параболические системы; конвекция-диффузия; тепломассоперенос; обратная задача; задача управления; краевая задача; корректность.

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