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MATHEMATICAL MODELLING OF WAVY SURFACE OF LIQUID FILM FALLING DOWN A VERTICAL PLANE AT MODERATE REYNOLDS' NUMBERS

L.A. Prokudina, South Ural State University, Chelyabinsk, Russian Federation, prokudina-la@mail.ru,
Ye.A. Salamatov, Emerson Process Management, Metran, Chelyabinsk, Russian Federation, salamatov@gmx.com, yevgeny.salamatov@emerson.com

Development of periodic disturbances on free surface of water film falling down vertical plane for Reynolds' number $Re \in [5;10]$ is investigated. The investigation is implemented in a scope of the nonlinear differential equation for evolution of free surface of falling down liquid film. The equation is solved by a finite differencies method at rectangular uniformly spaced grid. By researching the growth of unit inaccuracy, the conditions on parameters of computation grid for inaccuracies to be not increasing are obtained. As a result, waveforms of water film, time spent to form the regular wave mode and amplitudes of periodic disturbances are calculated. Calculated amplitudes and experimental ones are compared.

Keywords: liquid film; amplitude; waveform; nonlinear evolution of disturbances.

Introduction

Liquid film flows (thin layers of viscous fluids) are implemented in many industries such as chemical, oil, energy and others. Advantages of liquid film flows are energy efficiency, greater product purity etc. These advantages are determined by highly extended contact line between streams moving and interacting each other. The contact line is formed by waves on free surface of liquid film.

Wavy flows of liquid films have been investigated since the middle of the 20^{th} century [1, 2]. Nevertheless they are still interesting [3–10]. Their study is requiered by both practical (due to variety of environments where liquid film flows are realised) and theoretical reasons (since film flows are described by nonlinear partial differential equations).

1. The Model

Consider a thin layer of wavy liquid film falling down a vertical smooth plane under the action of gravity. The liquid is supposed to be viscous, uniform and incompressible. It is interacting with a gas streaming in parallel to the plane. The gas creates constant tangential stress $\bar{\tau}$ on the free surface of the liquid film (Fig. 1). If the liquid and the gas are moving in one direction, cocurrent mode is implemented, otherwise countercurrent mode is realised.

The equation

$$\begin{aligned}\frac{\partial\psi}{\partial t} &= b_1\frac{\partial^4\psi}{\partial x^4} + b_2\frac{\partial^2\psi}{\partial x^2} + b_3\frac{\partial\psi}{\partial x} + b_4\frac{\partial^2\psi}{\partial x\partial t} + b_5\psi\frac{\partial\psi}{\partial x} + b_6\psi\frac{\partial^2\psi}{\partial x^2} + b_7\psi\frac{\partial^4\psi}{\partial x^4} + b_8\psi\frac{\partial^2\psi}{\partial x\partial t} + \\ &+ b_9\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t} + b_{10}\left(\frac{\partial\psi}{\partial x}\right)^2 + b_{11}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial x^3} + b_{12}\psi^2\frac{\partial\psi}{\partial x} + b_{13}\psi^2\frac{\partial^2\psi}{\partial x^2} + b_{14}\psi\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial x^3}\end{aligned} \quad (1)$$

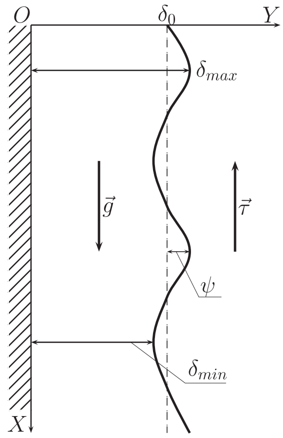

Fig. 1. Definition sketch

describes evolution of liquid film free surface [10–12]. Here $b_1 = -\dfrac{\sigma Re}{3}$, $b_2 = \dfrac{3}{40}Re^3F(F-\tau)$, $b_3 = -Re(F-\tau)$, $b_4 = \dfrac{5}{24}Re^2F$, $b_5 = -2ReF + Re\tau$, $b_6 = -\dfrac{3}{8}Re^3F\tau + \dfrac{9}{20}Re^3F^2$, $b_7 = 3b_1$, $b_8 = 4b_4$, $b_9 = b_8$, $b_{10} = b_6$, $b_{11} = 3b_1$, $b_{12} = -ReF$, $b_{13} = -\dfrac{3}{4}Re^3F\tau + \dfrac{9}{8}Re^3F^2$, $b_{14} = 6b_1$, Re is the Reynolds number, F is the Froude number, σ is the surface tension (unitless), τ is the shear stress (unitless). The required function $\psi(x,t)$ denotes the displacement of free surface (Fig. 1) from its undisturbed position δ_0 ($\psi \ll \delta_0$). We assume $\tau > 0$ for countercurrent mode and $\tau < 0$ for cocurrent one. If $\tau = 0$, free mode is implemented.

2. Searching for Periodical Solution to the Governing Equation

A periodical solution to (1) is searched numerically by finite defferencies method. We consider a rectangular domain $\Omega = \{(x,t) : x_{begin} \leqslant x \leqslant x_{end}, 0 \leqslant t < t_{end}\}$. Let us cover Ω by a uniform grid (x_i, t_j). The grid has a step $\Delta x = \dfrac{x_{end} - x_{begin}}{N_x}$ along the spacial variable x and has a step Δt along the time variable t. In this case $x_i = x_{begin} + i\Delta x$, $t_j = j\Delta t$ ($i = 0, 1, \ldots N_x$; $j = 0, 1, 2, \ldots$). By ψ_i^j denote $\psi(x_i, t_j)$. The derivatives in (1) are approximated as follows

$$\left.\frac{\partial^4\psi}{\partial x^4}\right|_i^j \approx \frac{\psi_{i+2}^j - 4\psi_{i+1}^j + 6\psi_i^j - 4\psi_{i-1}^j + \psi_{i-2}^j}{\Delta x^4} = \frac{d_{4,i}^j}{\Delta x^4}, \tag{2}$$

$$\left.\frac{\partial^3\psi}{\partial x^3}\right|_i^j \approx \frac{\psi_{i+2}^j - 3\psi_{i+1}^j + 3\psi_{i-1}^j - \psi_{i-2}^j}{2\Delta x^3} = \frac{d_{3,i}^j}{\Delta x^3}, \tag{3}$$

$$\left.\frac{\partial^2\psi}{\partial x^2}\right|_i^j \approx \frac{\psi_{i+1}^j - 2\psi_i^j + \psi_{i-1}^j}{\Delta x^2} = \frac{d_{2,i}^j}{\Delta x^2}, \tag{4}$$

$$\left.\frac{\partial\psi}{\partial x}\right|_i^j \approx \frac{\psi_{i+1}^j - \psi_{i-1}^j}{2\Delta x} = \frac{d_{1,i}^j}{\Delta x}, \tag{5}$$

$$\left.\frac{\partial\psi}{\partial t}\right|_i^j \approx \frac{\psi_i^{j+1}-\psi_i^j}{\Delta t}, \tag{6}$$

$$\left.\frac{\partial^2\psi}{\partial x\partial t}\right|_i^j \approx \frac{1}{\Delta t}\left(\frac{\psi_{i+1}^{j+1}-\psi_i^{j+1}}{\Delta x}-\frac{\psi_{i+1}^j-\psi_i^j}{\Delta x}\right). \tag{7}$$

Substituting (2) – (7) in to (1) and modifying it, we obtain

$$\psi_i^{j+1}\left(\frac{1}{\Delta t}+\frac{b_4}{\Delta t\Delta x}+\frac{b_8\psi_i^j}{\Delta t\Delta x}-\frac{b_9 d_{1,i}^j}{\Delta t\Delta x}\right)+\psi_{i+1}^{j+1}\left(-\frac{b_4}{\Delta t\Delta x}-\frac{b_8\psi_i^j}{\Delta t\Delta x}\right)=$$
$$=b_1\frac{d_{4,i}^j}{\Delta x^4}+b_2\frac{d_{2,i}^j}{\Delta x^2}+b_3\frac{d_{1,i}^j}{\Delta x}+b_5\psi_i^j\frac{d_{1,i}^j}{\Delta x}+b_6\psi_i^j\frac{d_{2,i}^j}{\Delta x^2}+b_7\psi_i^j\frac{d_{4,i}^j}{\Delta x^4}+b_{10}\frac{\left(d_{2,i}^j\right)^2}{\Delta x^2}+$$
$$+b_{11}\frac{d_{1,i}^j}{\Delta x}\frac{d_{3,i}^j}{\Delta x^3}+b_{12}\left(\psi_i^j\right)^2\frac{d_{1,i}^j}{\Delta x}+b_{13}\left(\psi_i^j\right)^2\frac{d_{2,i}^j}{\Delta x^2}+b_{14}\psi_i^j\frac{d_{1,i}^j}{\Delta x}\frac{d_{3,i}^j}{\Delta x^3}+\frac{\psi_i^j}{\Delta t}-$$
$$-\frac{b_4}{\Delta t}\frac{\psi_{i+1}^j-\psi_i^j}{\Delta x}-\frac{b_8\psi_i^j}{\Delta t}\frac{\psi_{i+1}^j-\psi_i^j}{\Delta x}-\frac{b_9 d_{1,i}^j\psi_i^j}{\Delta t\Delta x}, \qquad 0\leqslant i\leqslant N_x. \tag{8}$$

Assume

$$\Delta t=\eta\Delta x^3, \quad \eta>0, \tag{9}$$

and, multiplying both sides of (8) by $\eta\Delta x^4$, we get

$$\left(\alpha_i^j-\beta_i^j\right)\psi_i^{j+1}+\beta_i^j\psi_{i+1}^{j+1}=\eta\gamma_i^j+\left(\alpha_i^j-\beta_i^j\right)\psi_i^j+\beta_i^j\psi_{i+1}^j, \quad 0\leqslant i\leqslant N_x, \tag{10}$$

where $\alpha_i^j=\Delta x-b_9d_{1,i}^j$, $\beta_i^j=-b_4-b_8\psi_i^j$, $\gamma_i^j=b_1d_{4,i}^j+b_2d_{2,i}^j\Delta x^2+b_3d_{1,i}^j\Delta x^3+b_5\psi_i^jd_{1,i}^j\Delta x^3+b_6\psi_i^jd_{2,i}^j\Delta x^2+b_7\psi_i^jd_{4,i}^j+b_{10}\left(d_{1,i}^j\right)^2\Delta x^2+b_{11}d_{1,i}^jd_{3,i}^j+b_{12}\left(\psi_i^j\right)^2d_{1,i}^j\Delta x^3+b_{13}\left(\psi_i^j\right)^2d_{2,i}^j\Delta x^2+b_{14}\psi_i^jd_{1,i}^jd_{3,i}^j$.

Thus, in order to search a solution to (1) it is necessary to solve the system of equations (10).

Further, consider how computational inaccuracies behave while transferring from the j^{th} 'time layer' to the $(j+1)^{\text{th}}$ one. We will do that using a method of a unit inaccuracy growth as described in [13].

So, we assume that in any node (i,j) the function ψ_i^j contains inaccuracy ε_i^j (we suppose $\varepsilon_i^j\ll\psi_i^j$). But for the other nodes on the j^{th} 'time layer', we assume that the function ψ_i^j does not contain any inaccuracies (viz it is calculated exactly). For the node (i,j), it can be written that $\psi_i^j=\bar\psi_i^j+\varepsilon_i^j$, here $\bar\psi_i^j$ marks exactly calculated part of the function ψ (further, we denote by 'bar' any variable calculated exactly).

The inaccuracy ε_i^j leads to appearing of inaccuracies in the finite differencies (2) – (7) for the nodes $(i-2,j)$, $(i-1,j)$, (i,j), $(i+1,j)$, $(i+2,j)$ as follows

	$(i-2,j)$	$(i-1,j)$	(i,j)	$(i+1,j)$	$(i+2,j)$
d_1	$\bar d_{1,i-2}^j$	$\bar d_{1,i-1}^j+\frac{\varepsilon_i^j}{2}$	$\bar d_{1,i}^j$	$\bar d_{1,i+1}^j-\frac{\varepsilon_i^j}{2}$	$\bar d_{1,i+2}^j$
d_2	$\bar d_{2,i-2}^j$	$\bar d_{2,i-1}^j+\varepsilon_i^j$	$\bar d_{2,i}^j-2\varepsilon_i^j$	$\bar d_{2,i+1}^j+\varepsilon_i^j$	$\bar d_{2,i+2}^j$
d_3	$\bar d_{3,i-2}^j+\frac{\varepsilon_i^j}{2}$	$\bar d_{3,i-1}^j-\frac{3\varepsilon_i^j}{2}$	$\bar d_{3,i}^j$	$\bar d_{3,i+1}^j+\frac{3\varepsilon_i^j}{2}$	$\bar d_{3,i+2}^j-\frac{\varepsilon_i^j}{2}$
d_4	$\bar d_{4,i-2}^j+\varepsilon_i^j$	$\bar d_{4,i-1}^j-4\varepsilon_i^j$	$\bar d_{4,i}^j+6\varepsilon_i^j$	$\bar d_{4,i+1}^j-4\varepsilon_i^j$	$\bar d_{4,i+2}^j+\varepsilon_i^j$

(11)

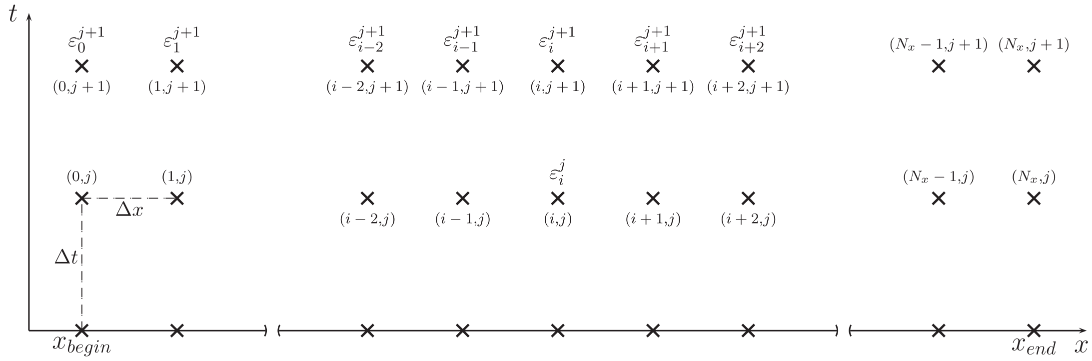

Fig. 2. The sketch of Ω

And then, inaccuracies (11) being on the j^{th} 'time layer' lead to inaccuracies on the $(j+1)^{\text{th}}$ 'time layer' as depicted on Fig. 2. For example, Equation (10) written for the node $(i+2, j+1)$ has the form

$$\left(\bar{\alpha}_{i+2}^j - \bar{\beta}_{i+2}^j\right)\left(\bar{\psi}_{i+2}^{j+1} + \varepsilon_{i+2}^{j+1}\right) + \bar{\beta}_{i+2}^j \bar{\psi}_{i+3}^{j+1} = \\ = \eta\left(\bar{\gamma}_{i+2}^j + b_1\varepsilon_i^j + b_7\bar{\psi}_{i+2}^j\varepsilon_i^j - b_{11}\bar{d}_{1,i+2}^j\frac{\varepsilon_i^j}{2} + b_{14}\bar{\psi}_{i+2}^j\bar{d}_{1,i+2}^j\frac{\varepsilon_i^j}{2}\right) + \\ + \left(\bar{\alpha}_{i+2}^j - \bar{\beta}_{i+2}^j\right)\bar{\psi}_{i+2}^j + \bar{\beta}_{i+2}^j\bar{\psi}_{i+3}^j. \quad (12)$$

By ε_{i+2}^{j+1} we denote the inaccuracy in the node $(i+2, j+1)$.

Taking into account $\psi_i^j \ll \delta_0$ and $\varepsilon_i^j \ll \psi_i^j$, modify Equation (12) as follows

$$\left(\Delta x + b_4\right)\varepsilon_{i+2}^{j+1} \approx \eta b_1 \varepsilon_i^j,$$
$$\varepsilon_{i+2}^{j+1} \approx \kappa_{i+2}\varepsilon_i^j,$$

and

$$\kappa_{i+2} = \eta\frac{b_1}{\Delta x + b_4}. \quad (13)$$

Similarly, missing manipulations due to their tedious, we find constants of proportionality for the nodes $(i+1, j+1)$, $(i, j+1)$, $(i-1, j+1)$, $(i-2, j+1)$:

$$\kappa_{i+1} = \eta\left(\frac{-4b_1 + b_2\Delta x^2 - \dfrac{b_3}{2}\Delta x^3}{\Delta x + b_4} + \frac{b_1 b_4}{(\Delta x + b_4)^2}\right), \quad (14)$$

$$\kappa_i = 1 + \frac{\eta\left(6b_1 - 2b_2\Delta x^2\right)}{\Delta x + b_4} + \frac{b_4}{\Delta x + b_4}\kappa_{i+1}, \quad (15)$$

$$\kappa_{i-1} = \frac{\eta\left(-4b_1 + b_2\Delta x^2 + \dfrac{b_3}{2}\Delta x^3\right) - b_4 + b_4\kappa_i}{\Delta x + b_4}, \quad (16)$$

$$\kappa_{i-2} = \frac{\eta\left(6b_1 - 2b_2\Delta x^2\right) + b_4\kappa_{i-1}}{\Delta x + b_4}. \quad (17)$$

Equations in (10) for the nodes $(l, j+1)$, $0 \leqslant l \leqslant i-3$, have the form

$$\left(\bar{\alpha}_l^j - \bar{\beta}_l^j\right)\left(\bar{\psi}_l^{j+1} + \varepsilon_l^{j+1}\right) + \bar{\beta}_l^j\left(\bar{\psi}_{l+1}^{j+1} + \varepsilon_{l+1}^{j+1}\right) = \eta\bar{\gamma}_l^j + \left(\bar{\alpha}_l^j - \bar{\beta}_l^j\right)\bar{\psi}_l^j + \bar{\beta}_l^j\bar{\psi}_{l+1}^j, \quad 0 \leqslant l \leqslant i-3.$$

Thus,

$$\varepsilon_l^{j+1} \approx \frac{b_4}{\Delta x + b_4}\varepsilon_{l+1}^{j+1}, \quad 0 \leqslant l \leqslant i-3,$$
$$\varepsilon_l^{j+1} \approx \frac{b_4}{\Delta x + b_4}\kappa_{l+1}\varepsilon_i^{j}, \quad 0 \leqslant l \leqslant i-3,$$
$$\kappa_l = \frac{b_4}{\Delta x + b_4}\kappa_{l+1}, \quad 0 \leqslant l \leqslant i-3. \tag{18}$$

The formulae (13) – (18) connect the unit inaccuracy ε_i^j with ones appearing on the $(j+1)^{\text{th}}$ "time layer". Obviously, the inaccuracies on the $(j+1)^{\text{th}}$ 'time layer' will not increase if parameters η and Δx are chosen so that $|\kappa_q| < 1$, $0 \leqslant q \leqslant i+2$. According to the formula (18), we get $|\kappa_0| < \cdots < |\kappa_l| < \cdots < |\kappa_{i-3}| < |\kappa_{i-2}|$. Therefore we may require only

$$|\kappa_q| < 1, \quad i-2 \leqslant q \leqslant i+2. \tag{19}$$

Unequalities (19) can be written with respect to η as follows

$$\eta < \frac{\Delta x + b_4}{|b_1|} = \eta_1,$$
$$\eta < \frac{(\Delta x + b_4)^2}{\left|3b_1b_4 + 4b_1\Delta x - b_2b_4\Delta x^2 + \left(4b_1 + \frac{b_3b_4}{2}\right)\Delta x^3 + \frac{b_3}{2}\Delta x^4\right|} = \eta_2$$
$$\eta < \frac{2(\Delta x + b_4)^3}{\left|3b_1b_4^2 + 8b_1b_4\Delta x + (6b_1 - b_2b_4^2)\Delta x^2 - \left(3b_2b_4 + \frac{b_3b_4^2}{2}\right)\Delta x^3 - \left(2b_2 + \frac{b_3b_4}{2}\right)\Delta x^4\right|} = \eta_3,$$
$$\eta < \frac{(\Delta x + b_4)^4}{\left|\begin{array}{l} b_1b_4^3 + 4b_1b_2^4\Delta x + 6b_1b_4\Delta x^2 + \left(4b_1 + \frac{b_3b_4^2}{2}\right)\Delta x^3 - (b_2b_4 + b_3b_4^2)\Delta x^4 - \\ \qquad - \left(b_2 + \frac{3}{2}b_3b_4\right)\Delta x^5 - \frac{b_3}{2}\Delta x^6 \end{array}\right|} = \eta_4,$$
$$\eta < \frac{(\Delta x + b_4)^5}{\left|\begin{array}{l} 5b_1b_4^4 + 20b_1b_4^3\Delta x + (30b_1b_4^2 - 2b_2b_4^4)\Delta x^2 - \left(20b_1b_4 - 8b_2b_4^3 - \frac{b_3b_4^3}{2}\right)\Delta x^3 + \\ + (6b_1 - 11b_2b_4^2 + b_3b_4^3)\Delta x^4 + \left(\frac{3}{2}b_3b_4^2 - 7b_2b_4\right)\Delta x^5 + \left(\frac{b_3b_4}{2} - 2b_2\right)\Delta x^6 \end{array}\right|} = \eta_5.$$

Finally, we obtain the next condition

$$\eta < \min(\eta_1, \eta_2, \eta_3, \eta_4, \eta_5). \tag{20}$$

3. Results

Considere an initial periodical disturbance

$$\psi(x, 0) = a_0 \cdot \cos(kx) \tag{21}$$

Table

Grid parameters

Re	k	Δx	$\eta \times 10^{-3}$
5	0,07246	0,270	1,11
6	0,08311	0,273	0,94
7	0,09332	0,276	0,78
8	0,10211	0,273	0,62
9	0,10976	0,282	0.49
10	0,11629	0,274	0,37

that is developed on free surface of water film falling down a vertical plane. Disturbance (21) has the amplitude $a_0 = 10^{-4} \ll 1$ and the wave number k (see Table). We consider $Re \in [5; 10]$ and $\tau = 0$.

Put $\eta = \dfrac{1}{3}\min(\eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$. The grid parameters are placed in Table. The length of $[x_{\min}; x_{\max}]$ is multiple of 30 wave lengths of (21). Calculations were carried out untill $t_{end} = 1000$.

Our calculations demonstrate, that the regular wavy flows are formed on free surface of vertical water film. The waveforms are shown on Fig. 3 (we illustrated only one period and the x axes was scaled as $\dfrac{2\pi}{k}$). Similar waveforms are observed many times in experiments, the reader may see them in [2, 7, 14, 15], for example.

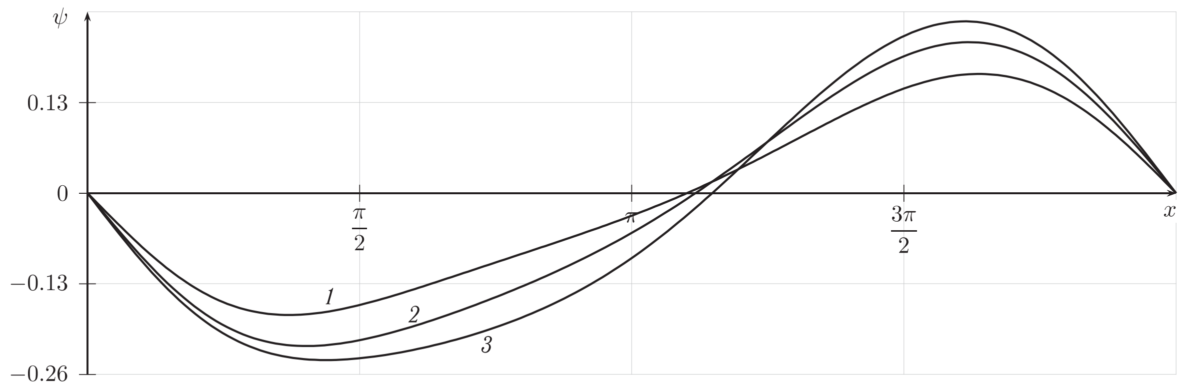

Fig. 3. Waveforms on free surface of water film: – $Re = 5$; 2 – $Re = 7$; 3 – $Re = 10$

The higher Reynolds' number Re, the higher wave amplitude is (Fig. 4, 5). Wave amplitudes depicted on Fig. 4 were calculated by formula [1, 2]

$$a = \frac{\delta_{\max} - \delta_{\min}}{\delta_{\max} + \delta_{\min}}. \tag{22}$$

We used the formula [14, 16]

$$A = \frac{\delta_{\max} - \langle\delta\rangle}{\langle\delta\rangle} \tag{23}$$

to calculate wave amplitudes depicted on Fig. 5. By $\langle\delta\rangle$ in (23) the mean liquid film thickness is denoted.

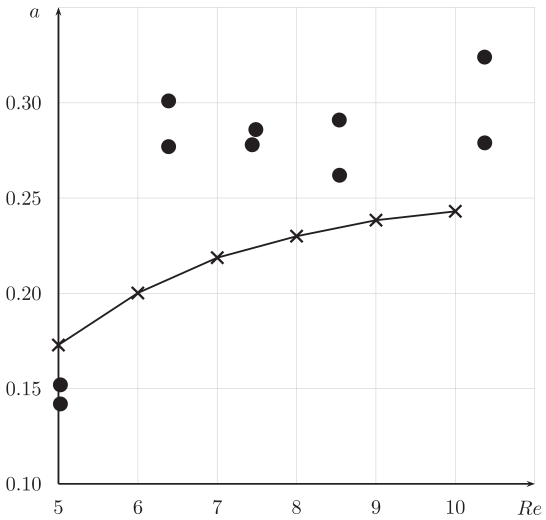

Fig. 4. Amplitudes (water) calculated by (22): • – [2]; × – our results

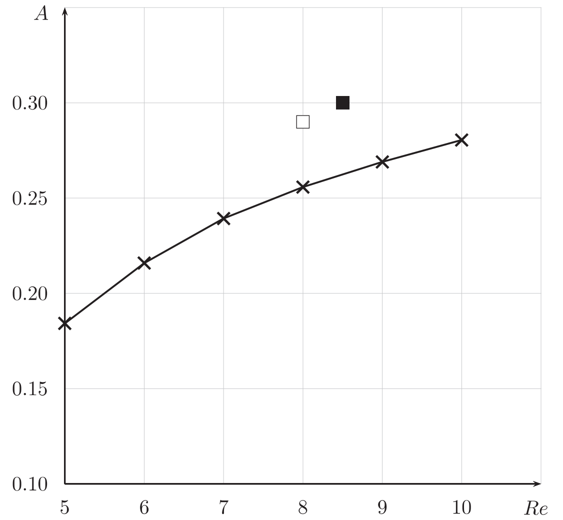

Fig. 5. Amplitudes (water) calculated by (23): ■ – [14]; □ – [16]; × – our results

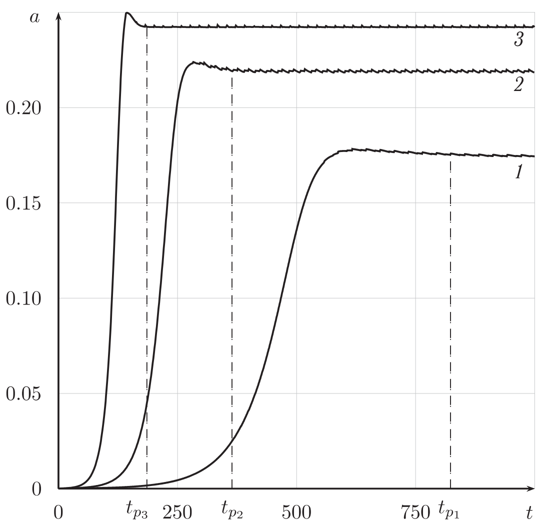

Fig. 6. Amplitude time dependence for water film: 1 – $Re = 5$; 2 – $Re = 7$; 3 – $Re = 10$

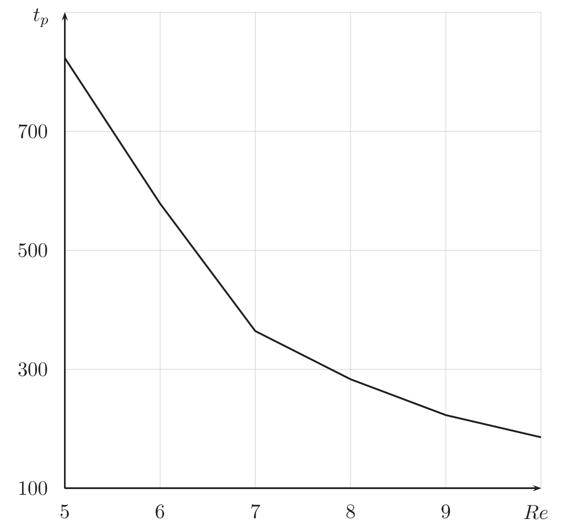

Fig. 7. Duration of regular wave forming for water film

Fig. 6 shows how amplitudes change while regular wavy flows are forming. On Fig. 6 we marked the times $t_{p_1} = 823,2$, $t_{p_2} = 364,3$, $t_{p_3} = 185,7$ when the regular wave flows had already formed. As shown on Fig. 7, the higher Reynolds' number Re, the less time t_p is required.

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МОДЕЛИРОВАНИЕ ВОЛНОВОЙ ПОВЕРХНОСТИ ВЕРТИКАЛЬНОЙ ЖИДКОЙ ПЛЕНКИ ПРИ УМЕРЕННЫХ ЧИСЛАХ РЕЙНОЛЬДСА

Л.А. Прокудина, Е.А. Саламатов

В рамках модельного нелинейного дифференциального уравнения, которое описывает эволюцию свободной поверхности вертикальной жидкой пленки, исследовано развитие периодических возмущений на вертикальной пленке воды в диапазоне чисел Рейнольдса $Re \in [5; 10]$. Модельное уравнение решалось методом конечных разностей на прямоугольной равномерной сетке. Исследованием роста единичной погрешности получены условия, которым должны удовлетворять параметры расчетной сетки, чтобы погрешности не увеличивались в ходе вычислений. В результате рассчитаны профили волн на свободной поверхности вертикальной пленки воды, время формирования режима стекания с регулярным профилем волны, характер изменения амплитуды периодического возмущения при формировании режима с регулярным профилем волны. Осуществлено сравнение рассчитанных амплитуд с результатами экспериментов.

Ключевые слова: жидкая пленка; амплитуда; профиль волны; нелинейное развитие возмущений.

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Людмила Александровна Прокудина, доктор физико-математических наук, доцент, кафедра «Прикладная математика», Южно-Уральский государственный университет (г. Челябинск, Российская Федерация), prokudina-la@mail.ru.

Евгений Александрович Саламатов, кандидат физико-математических наук, Emerson Process Management, ЗАО «ПГ«Метран» (г. Челябинск, Российская Федерация), salamatov@gmx.com, yevgeny.salamatov@emerson.com.

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